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  - Signal analysis versus system analysis
  - Basic method for signal analysis
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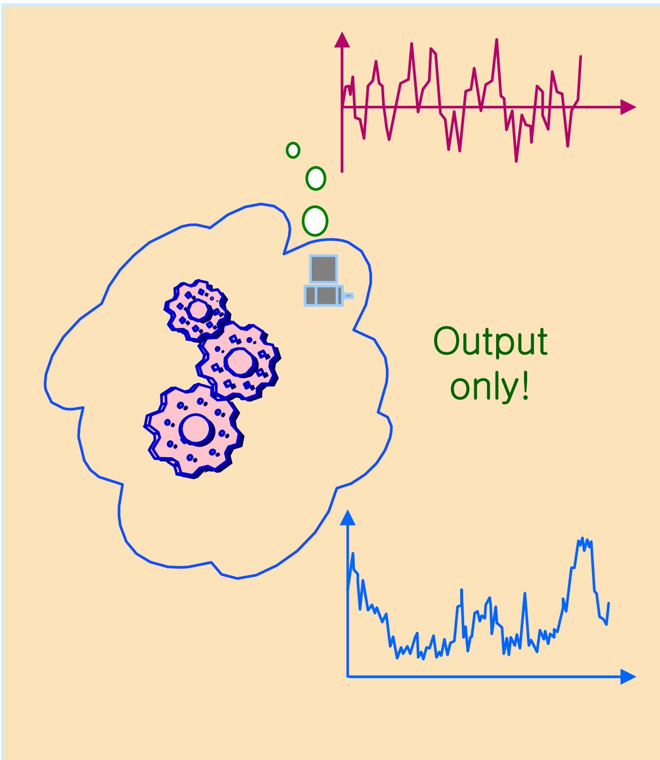
  - Non-stationary signal analysis

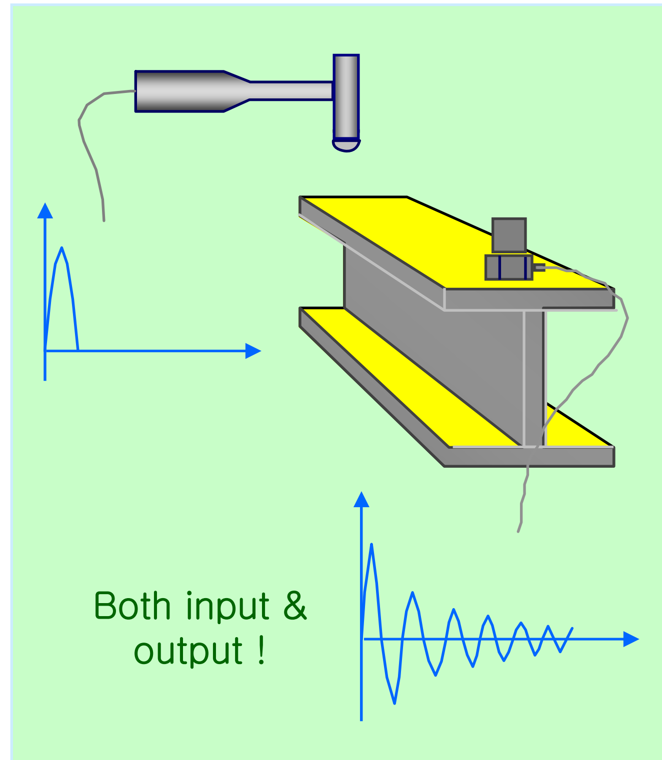
- Signal analysis ver System analysis
  - Signal analysis
    - analysis for running machine
  - System analysis
    - analysis unique feature for holding machine

- Signal analysis & System analysis

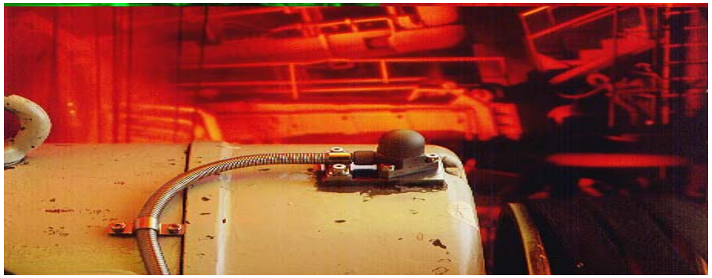
Signal analysis



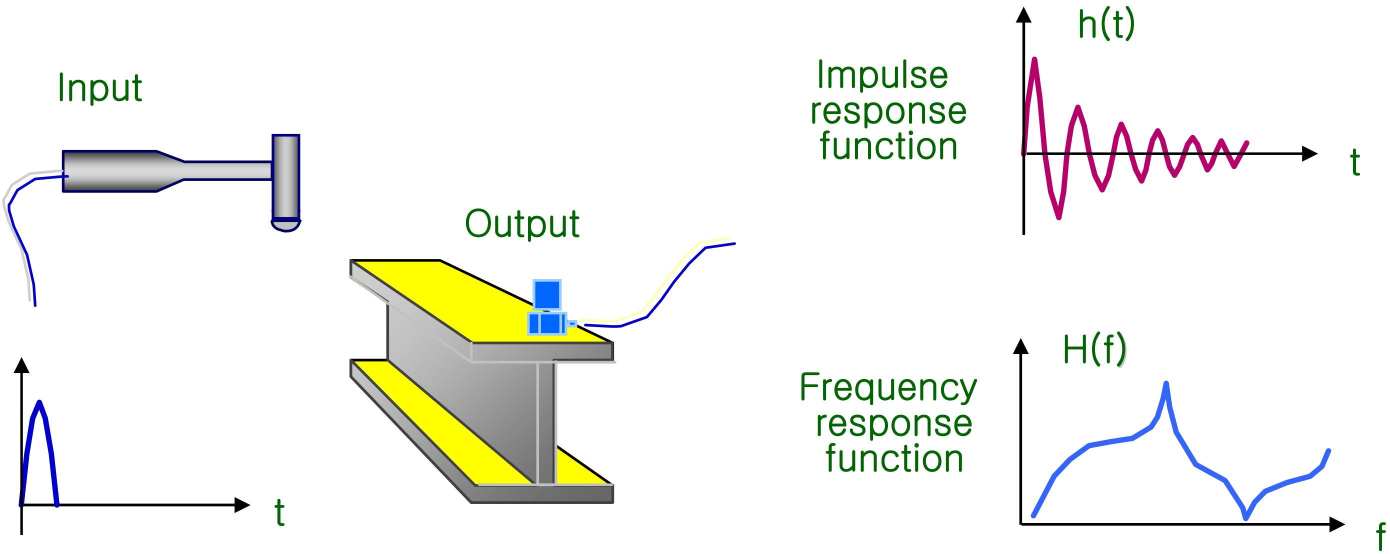
System analysis



- Signal Analysis
  - Analysis for output signal only
  - Application
    - impossible condition to measure input signal
      - ex) machine condition diagnosis and monitoring
    - impossible condition not to separate source
      - ex) Environmental noise
    - focus on output signal only
      - ex) Balancing



- System Analysis

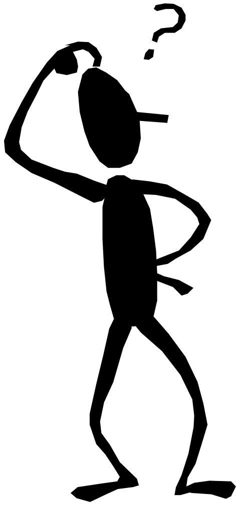


- Simultaneous measure both input and output ⇒ Phase relation
- h(t), H(f) ⇒ system character
  ⇒ Predict system behavior for external excitation

4.1 Signal Analysis & Functions

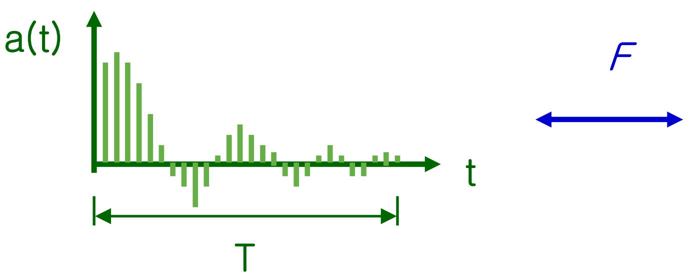
- The basic method of signal analysis

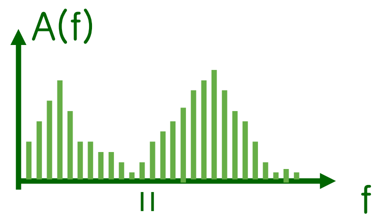
  - Signal type
    - Stationary, Non-stationary
  - Unit
    - RMS, PWR, PSD, ESD, Linear, dB…
  - Average signal
    - Lin, Exp, Peak…
  - Expression
    - Time, Frequency, Time & Frequency…
  - Measurement credit
    - Meas. error, Dynamic range, S/N ratio …
  - Triggering
    - Free Run, Internal, External…



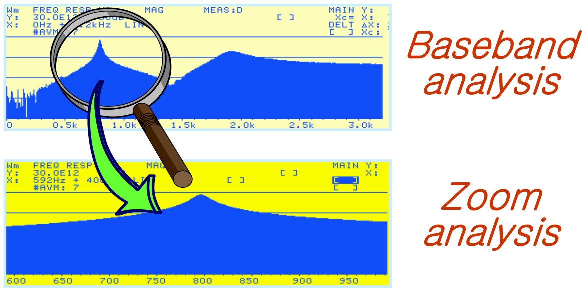
- Zoom analysis (Frequency resolution of FFT)

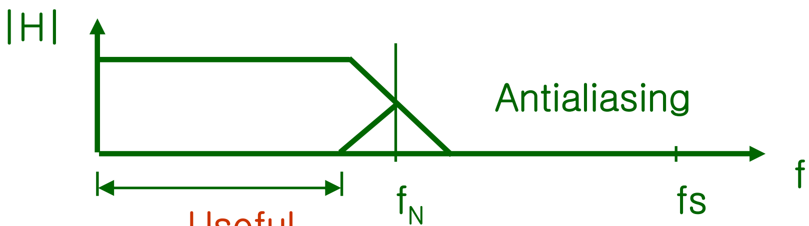
  : Reduce Leakage, Picket fence effect error.
  Increase frequency resolution.





$$\Delta f = \frac{1}{T}$$





Useful
Frequency range : $f_{span} = N \times \Delta f$

$$T \cdot \Delta f = 1$$

$N = 2048$

$f_s = 2.56 \times f_{span}$

➡ $\Delta f = \frac{f_s}{2048} = \frac{f_{span}}{800}$

$N_{span} = 800$

f_{span} : Max freq. span without distortion

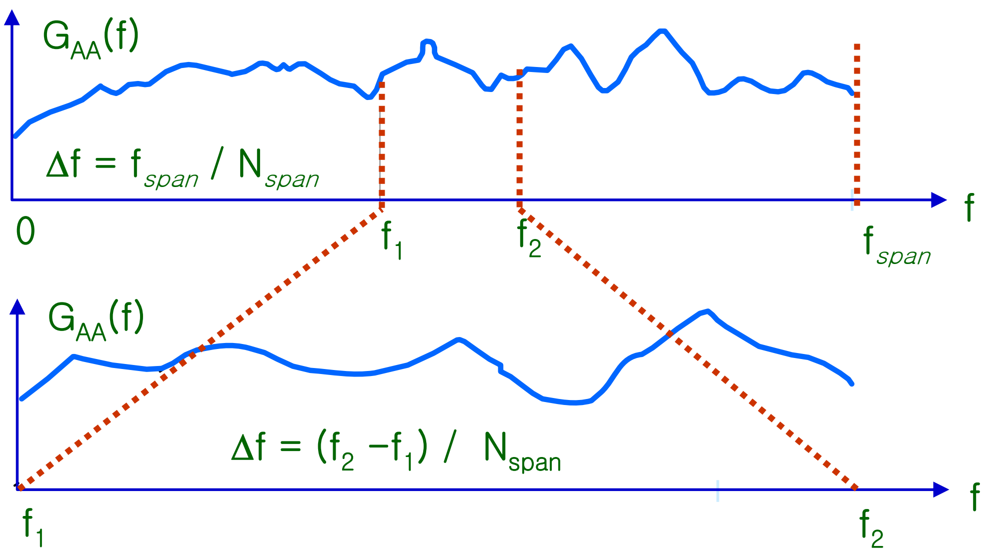
N_{span} : No. of lines in f_{span}

Example : (N_{span} = 800 lines)

T[s]	Δf [Hz]	f_{span} [Hz]
512	0.00195	1.56
–	–	–
2	0.5	400
1	1	800
0.5	2	1600
–	–	–
0.031	32	25600

- Zoom analysis (Real-time zoom)

  - Apply available resolution $\Delta f = f_{span} / N_{span}$
  - Move frequency span from $0 \rightarrow f_{span}$ to $f_1 \rightarrow f_2$



the sampling interval

$\Delta t = 1 / f_s$

and record length

$T = N \Delta t = N / f_s$

thus, analysis resolution

$f = 1 / T = f_s / N$

Reducing the resolution Δf ,

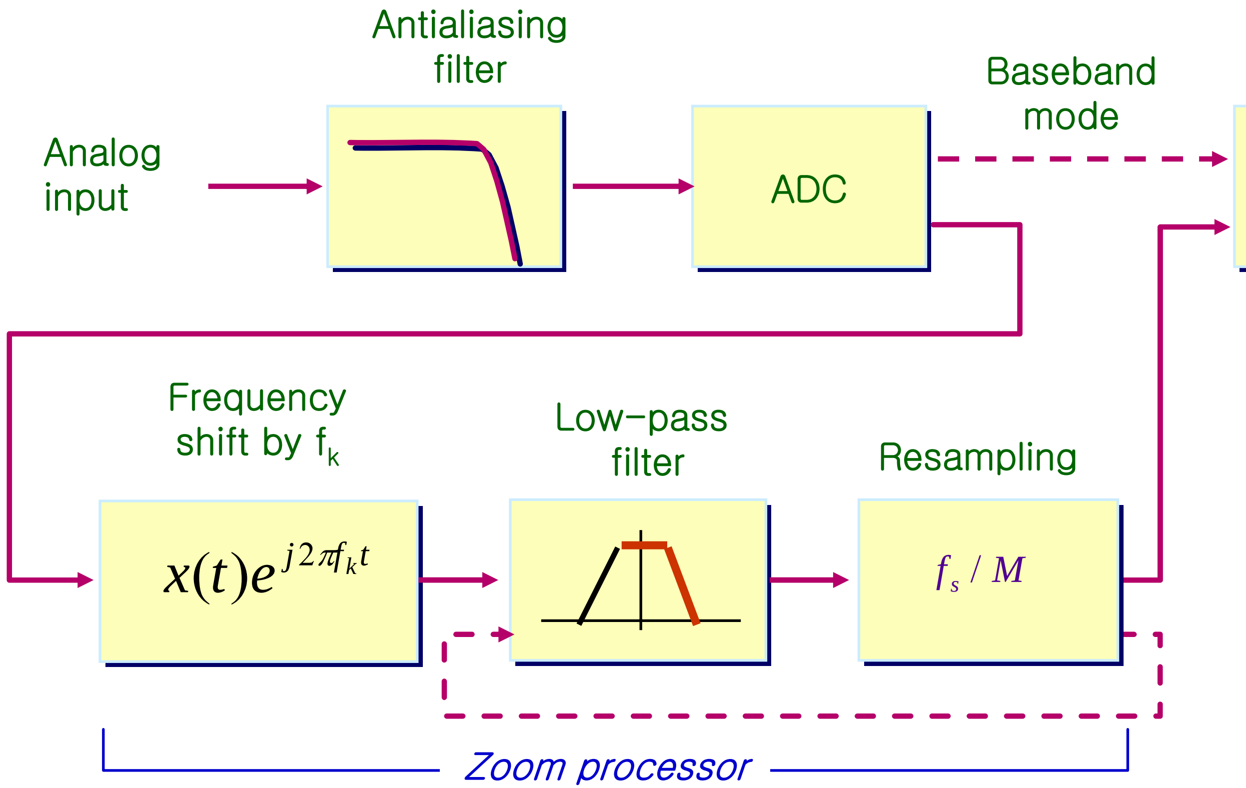
1) reduce the sampling frequency f_s

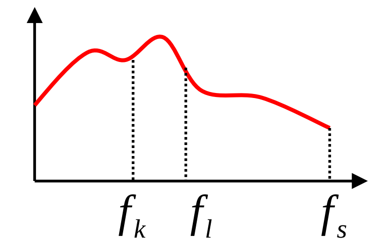
“ Real-time zoom “

2) increased the record length N

“ non-destructive zoom “

- Zoom analysis (Real-time Zoom)





$$f_s' = B = f_l - f_k \qquad \frac{f_s}{B} = M$$

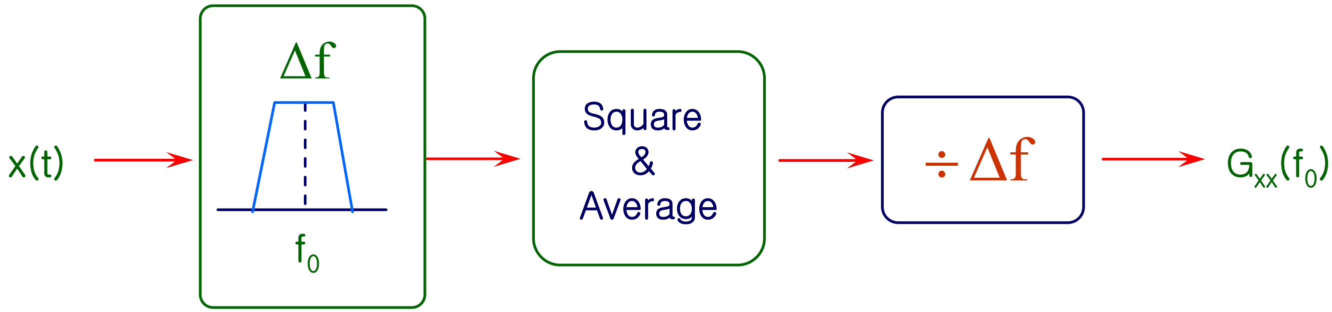
$$h = \frac{1}{f_s} \qquad h' = \frac{M}{f_s}$$

$$V(f) = \int_{-\infty}^{\infty} x(t)e^{j2\pi f_k t}e^{-j2\pi f t}dt = \int_{-\infty}^{\infty} x(t)e^{j2\pi(f-f_k)t}dt = X(f-f_k)$$

f_k : modulation frequency

The extra length of the time signal required to obtain the finer resolution is achieved by reducing the sampling rate.

- Power Spectrum
  - Energy which is divided by filter bandwidth
  - Unit
    - RMS : Deterministic signal
    - PSD : Random signal
    - ESD : Transient signal



Auto PSD : $S'_{xx}(f) = \dfrac{X^*(f) \bullet X(f)}{T} = \Delta f \; X^*(f) \bullet X(f)$

- Definitions of PSD & CSD

  - Average Power for random signal $x(t)$

$$P_{avg} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt$$

  - PSD in random process

$$S_{xx}(f) = \lim_{T\to\infty} < \frac{|X(f,T)|^2}{T} > = \lim_{T\to\infty} \frac{1}{T} E[|X_k(f,T)|^2]$$

  - Cross Spectral Density Function

$$S_{xy}(f) = \lim_{T\to\infty} < \frac{X^*(f,T)Y(f,T)}{T} > = \lim_{T\to\infty} \frac{1}{T} E[X_k^*(f,T)Y_k(f,T)]$$

Properties

$i)\quad S_{xy}(f)\quad is\ a\ complex\ function$

$ii)\quad S_{xy}(-f) = S_{xy}^*(f) = S_{yx}(f)$: By definition of Fourier transform

$iii)\quad if\ x = y\quad S_{xx}(f)\quad is\ positive\ real\qquad S_{xx}(-f) = S_{xx}(f)\quad even\ fn$

- Wiener–Khinchine Relations

$$S_{xf}(f) = \int_{-\infty}^{\infty} R_{xy}(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

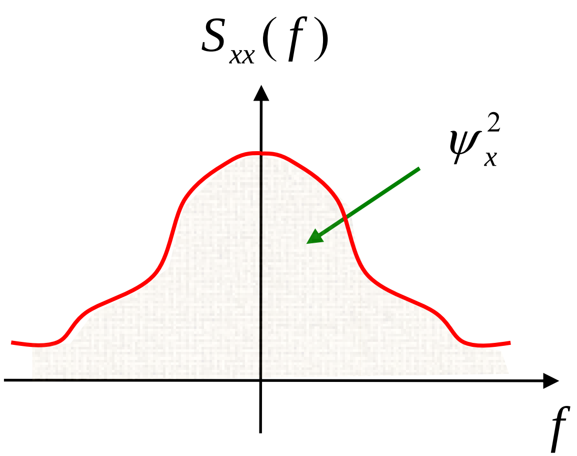
$$R_{xf}(\tau) = \int_{-\infty}^{\infty} S_{xy}(f)\, e^{j2\pi f\tau}\, df$$

$$if\ \ x(t) = y(t) \qquad S_{xy}(f) \rightarrow S_{xx}(f) \qquad R_{xy}(\tau) \rightarrow R_{xx}(\tau)$$

Properties

i) $R_{xx}(o) = \psi_x^2$ *mean square*

$$= \int_{-\infty}^{\infty} S_{xx}(f)df$$



- Proof of Wiener-Khinchine Relations
  - Cross Correlation Function

$$R_{xy}(\tau) = E[x(t)y(t+\tau)] = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)y(t+\tau)dt$$

Fourier Transform of $R_{xy}(\tau)$

$$\int_{-\infty}^{\infty} R_{xy}(\tau)e^{-j2\pi f\tau}d\tau = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} x(t)y(t+\tau)dt\, e^{-j2\pi f\tau}d\tau$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} x(t)y(t+\tau)e^{-j2\pi f\tau}dt\, d\tau$$

Let $\tau = s - t$

$d\tau = ds$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} x(t)y(s)e^{-j2\pi f(s-t)}ds\, d\tau$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{j2\pi ft}\int_{-T/2}^{T/2} y(s)e^{-j2\pi fs}ds\, d\tau$$

$$= \lim_{T\to\infty}\frac{1}{T}X^*(f)Y(f) = S_{xf}(f)$$

: Cross Spectral Density Function !!

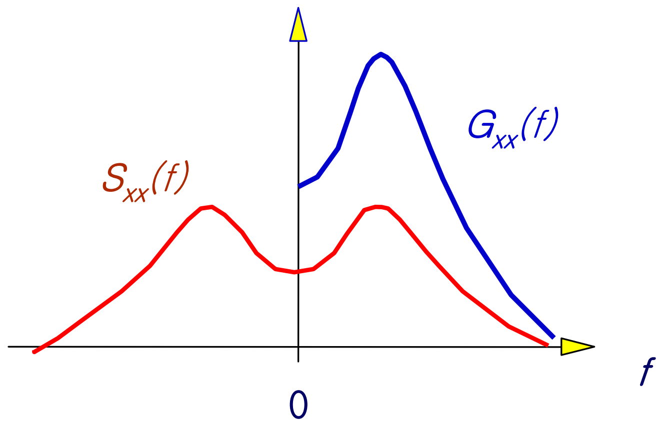
- Power spectrum

  - Spectrum calculation of FFT

Auto PSD : $S'_{xx}(f) = \dfrac{X^*(f) \bullet X(f)}{T}$

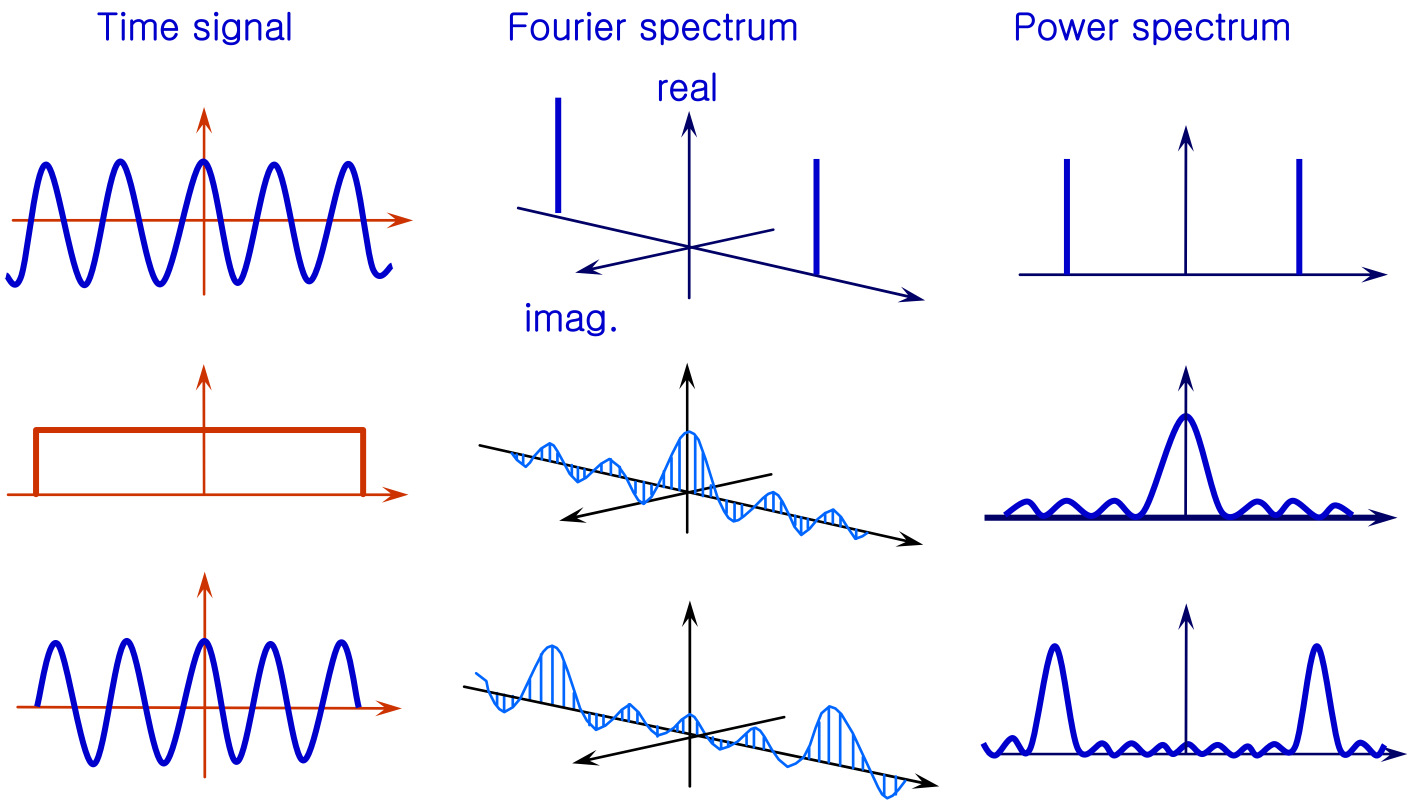
$$S_{xx}(f) = X^*(f) \bullet X(f)$$: 2−sided spectrum

$$G_{xx}(f) = \begin{cases} 2S_{xx}(f), & f > 0 \\ S_{xx}(f), & f = 0 \\ 0, & f < 0 \end{cases}$$: 1−sided spectrum

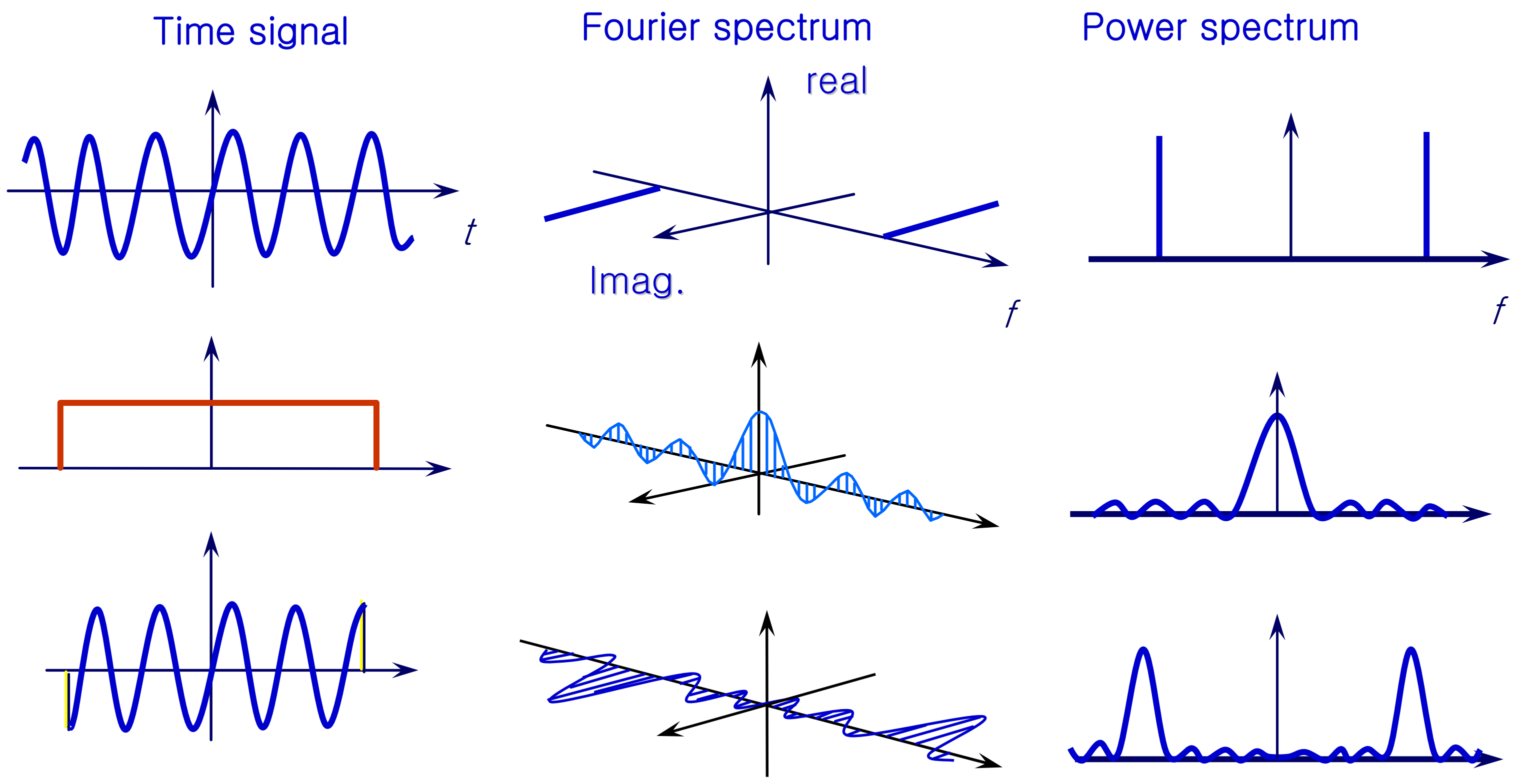


- X(f) : (instantaneous) Fourier transform
- X*(f) : complex conjugate of X(f)

- Power Spectrum of Cosine



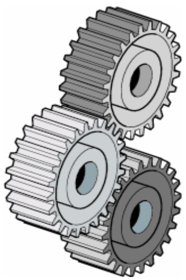
- Power Spectrum of Sine

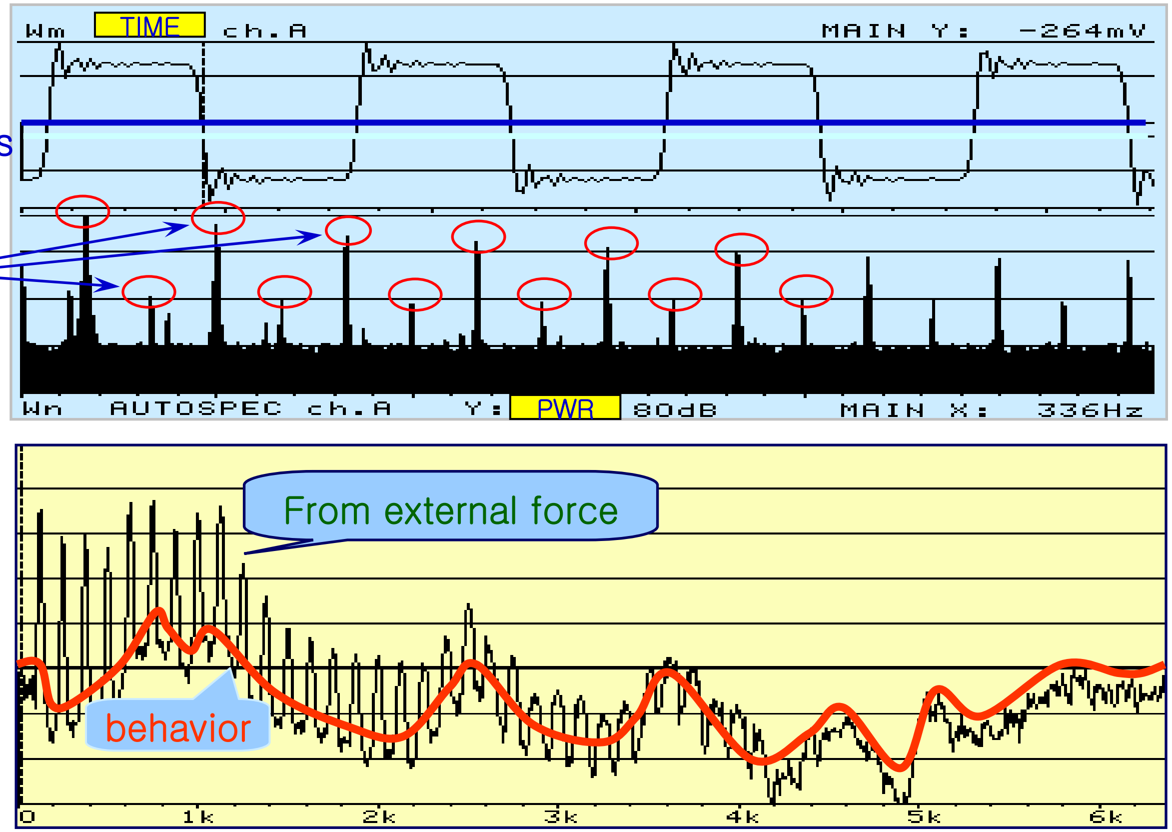


- Power spectrum of a periodic signal

There are
harmonic components

harmonics





- Cross Power Spectrum(I)

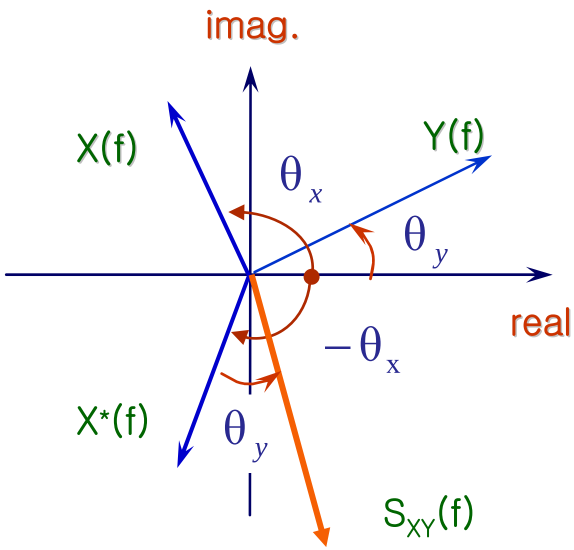
$S_{XY}(f)$: cross spectrum from Y to X

$$S_{xx}(f) = X^*(f) \bullet X(f)$$
$$= F[R_{xy}(\tau)]$$

$$X(f) = |X(f)| e^{j\theta_x(f)}$$

$$Y(f) = |Y(f)| e^{j\theta_y(f)}$$

- Amplitude : $|X(f) \cdot Y(f)|$
- Phase : $\theta_y(f) - \theta_x(f)$



- Cross Power Spectrum(II)

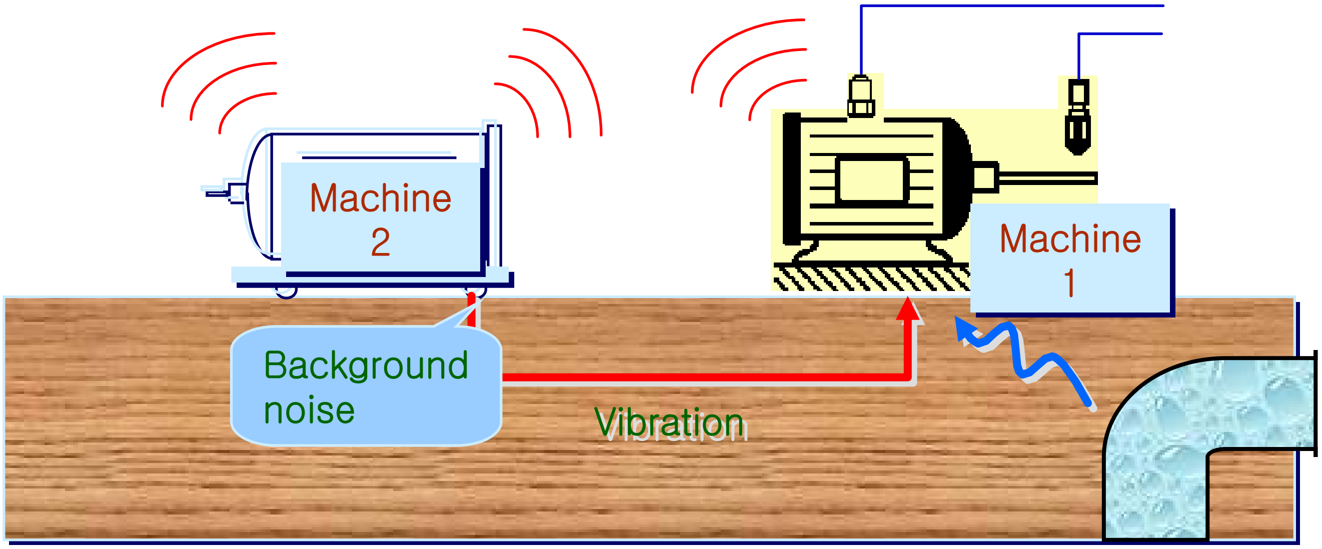
  - Amplitude $|S_{xy}(f)|$: correlation of x and y signal
  - Phase $\angle S_{xy}(f)$: Phase difference (function of frequency)
    - Phase of $S_{xy}(f)$: $\theta_y(f) - \theta_x(f)$, Phase of $S_{yx}(f)$: $\theta_x(f) - \theta_y(f)$
  - $|S_{xy}(f)| = |S_{yx}(f)|$, $S_{xy}(f) = S^*_{yx}(f)$

  - Using of other function calculation
    - Frequency Response Function, Cross Correlation,
    - Coherence, Sound Intensity estimation
  - Analysis of two signal phase relation
    - propagation delay
  - Analysis of two signal correlation
    - cf) FRF

- Signal Enhancement (Time Averaging)

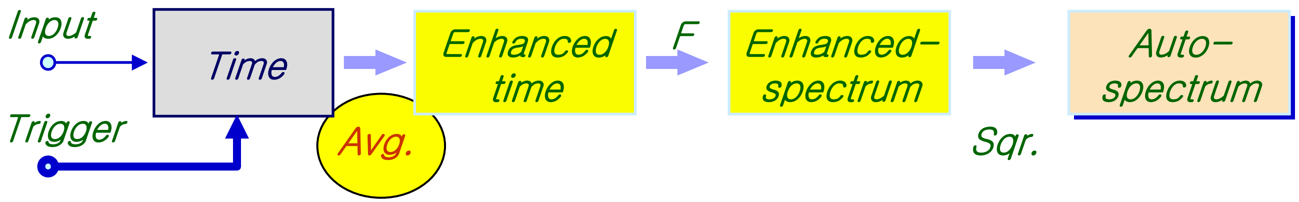
  - Extraction of a periodically repeating signal from additive contaminating noise
  - Applications :
    - Wave form analysis
    - Reduce background noise
    - Enhance orders
    - Separation of mechanical & electrical vibrations

- Signal Enhancement Application Example



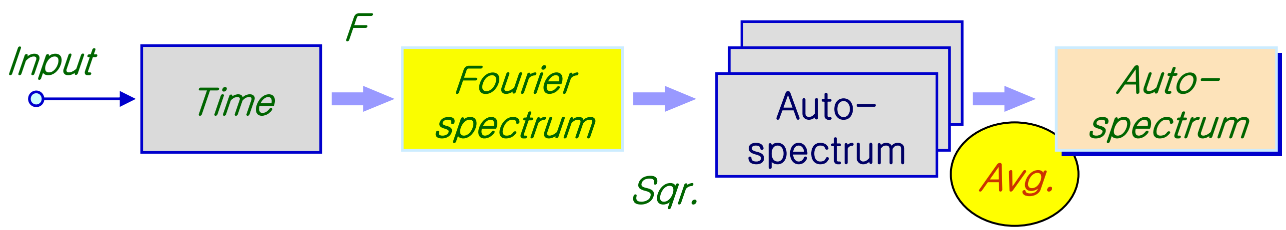
- Signal enhancement in machine analysis
- Vibration measurement on a machine shop floor

- Time vs Frequency Domain Averaging



- Synchronous time domain averaging
  - Requires a synchronous trigger signal
  - Fast averaging (no FFT needed)

The main feature of signal enhancement is that the averaging process takes place in the time domain



- Frequency domain averaging

- Signal Enhancement Theory I (Time domain)

  - Time signal

$$x(t) = a(t) + m(t)$$

- a(t) = Periodic signal
- m(t) = Random noise signal
- x(t) = Measured signal

  - Time domain averaging

$$\overline{x(t)} = \frac{1}{N_A} \cdot \sum \left[a(t) + m(t) \right]$$

$$= \frac{1}{N_A} \cdot \left[N_A \cdot a(t) + \sqrt{N_A} \cdot m(t) \right]$$

$$= \overline{a(t)} + \frac{1}{\sqrt{N_A}} \cdot m(t)$$

Time records averaged N_A times will add coherently (amplitude summation), while the uncorrelated noise will add incoherently (power summation).

- Basic Concepts

$m(t)$: random noise with mean μ_m

Mean square error of the sample mean $\overline{m(t)}$

$$\psi_{mm}^2 = E[(\overline{m} - \mu_m)^2] = E\left[\left(\frac{1}{N_A}\sum_{i=1}^{N_A} m_i - \mu_m\right)^2\right] = \frac{1}{N_A^2} E\left[\sum_{i=1}^{N_A}(m_i - \mu_m)^2\right]$$

$$= \frac{1}{N_A^2} N_A \sigma_m = \frac{\sigma_m}{N_A} \qquad \therefore \ \psi_{mm} = \frac{\sigma_m}{\sqrt{N_A}}$$

$$\mu_m = 0 \ \rightarrow \ \psi_{mm} = \frac{\psi_s}{\sqrt{N_A}}$$

ψ_s^2 : Mean square value of the sample

- Basic Concepts

  - Non-synchronous trigger signal

$$x(t)\ averaging \quad \equiv \quad X(f)\ averaging$$

If the trigger is not synchronized, both data, x(t) and X(f), will be distorted.
Therefore, in this case, the averaging of power spectrum is only valid.

- Signal Enhancement Theory I (Time domain)

  - *Enhanced auto-spectrum*

$$G_{\overline{X}\,\overline{X}} = \overline{X(f)} \cdot \overline{X^*(f)}$$

$$= \left(\overline{A(f)} + \frac{1}{\sqrt{N_A}} \cdot M(f) \right) \cdot \left(\overline{A(f)} + \frac{1}{\sqrt{N_A}} \cdot M(f) \right)^*$$

$$= \overline{A(f)} \cdot \overline{A^*(f)} + \frac{1}{N_A} \cdot M(f) \cdot M^*(f)$$

$$= G_{\overline{A}\,\overline{A}} + \frac{1}{N_A} \cdot G_{MM}$$

A(t) and M(t) are mutually uncorrelated !!

  - *Frequency domain averaging, Auto-spectrum*

$$G_{XX} = G_{AA} + G_{MM}$$

G_{mm} is not decreased by averaging method because it is power that has positive real values.

4.1 Signal Analysis & Functions

- Signal Enhancement Theory I (Time domain)

< Signal to Noise Ratio >

- Time averaging

$$G_{\bar{X}\bar{X}} = G_{\bar{A}\bar{A}} + \frac{1}{N_A} \cdot G_{\bar{M}\bar{M}}$$

- Frequency domain averaging

$$G_{XX} = G_{AA} + G_{MM}$$

- Signal to noise improvement

$$S/N \approx N_A \qquad S/N = 10\log\left(\frac{G_{AA}}{1/N_A G_{MM}}\right)$$

Equals to number of averages

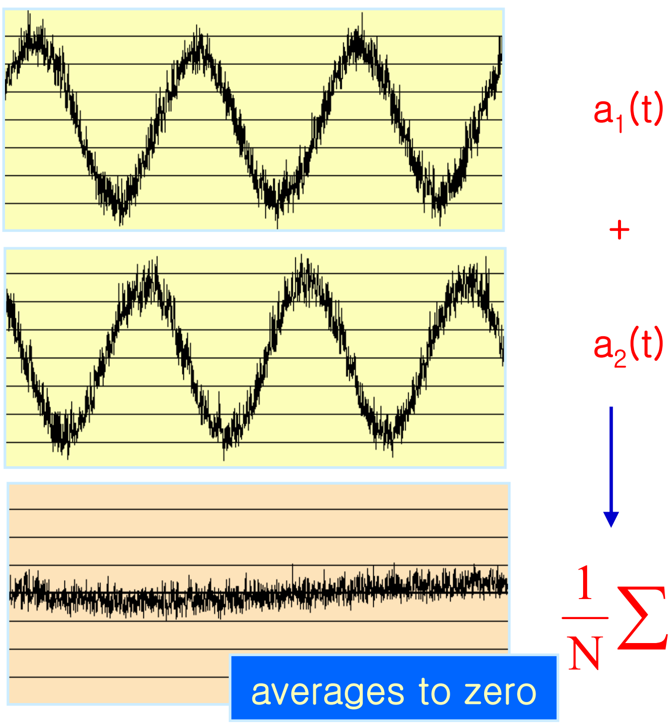
$$S/N = 10\log N_A \quad [dB]$$

N_A	$10\log_{10}(N_A)$
10	10 dB
100	20 dB
1,000	30 dB
10,000	40 dB
32,767	45 dB

* m(t) : uncorrelated random noise

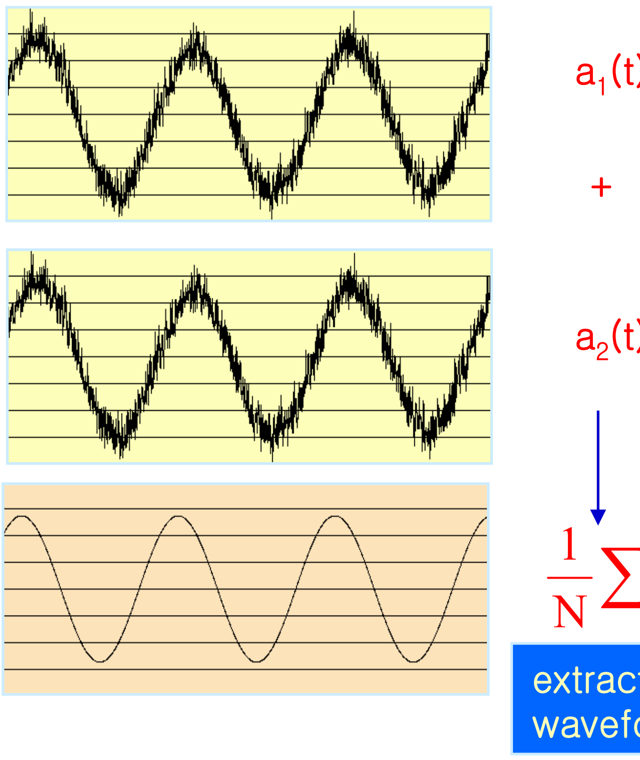
- Synchronous Time Averaging

  - Time Averaging no Trigger



$$\frac{1}{N}\sum a_i(t)$$

  - Synchronous Time Averaging

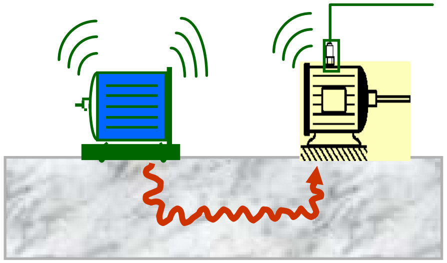


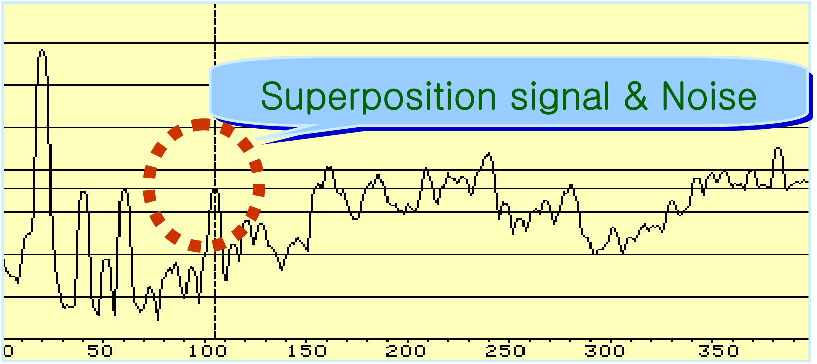
$$\frac{1}{N}\sum a_i(t)$$

The difference between non-synchronous time averaging and synchronous time averaging is illustrated in Figures.

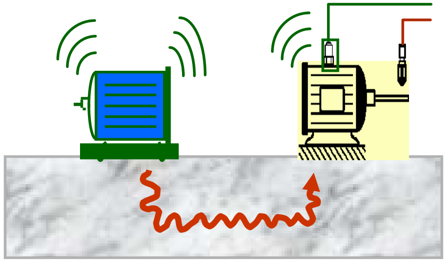
- Application of Signal Enhancement I

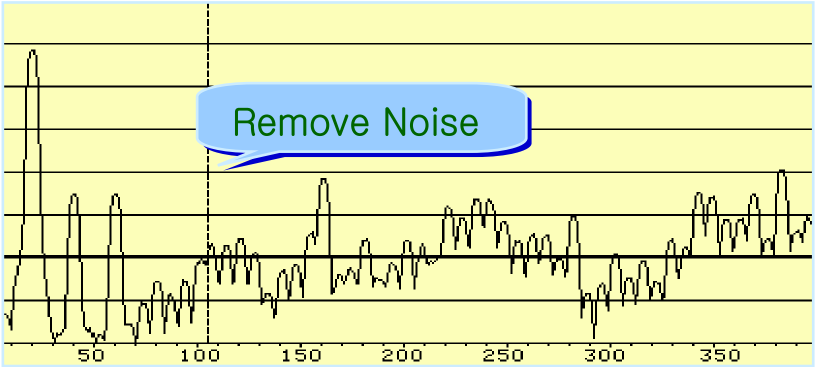
- Frequency averaging, Autospectrum





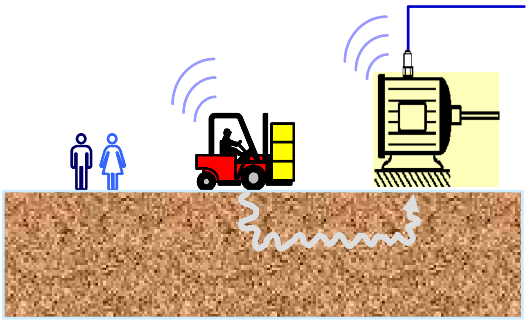
- Time averaging, Autospectrum



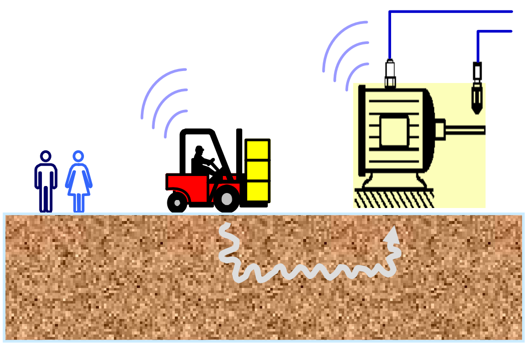


4.1 Signal Analysis & Functions

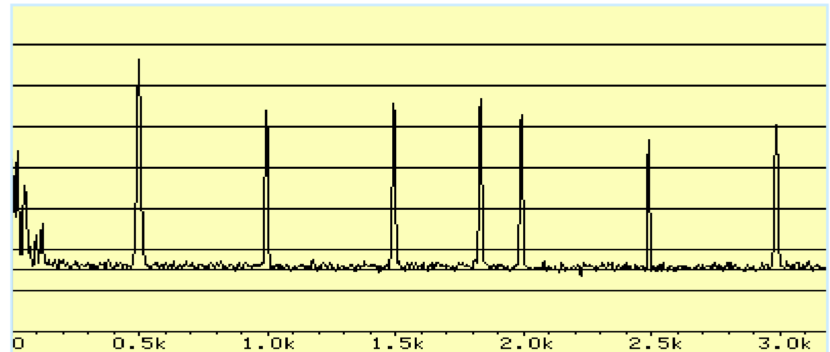
- Application of Signal Enhancement II



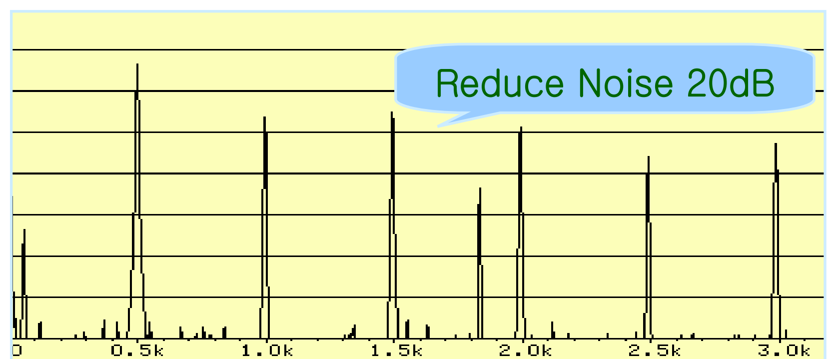
- N_A = 100 averages
- S/N improvement of 20dB



- Frequency averaging, Auto-spectrum



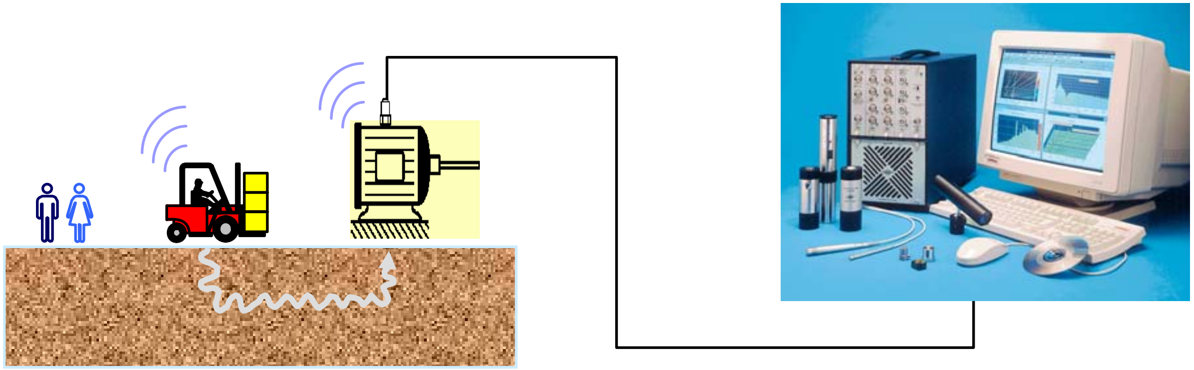
- Time averaging, Auto-spectrum



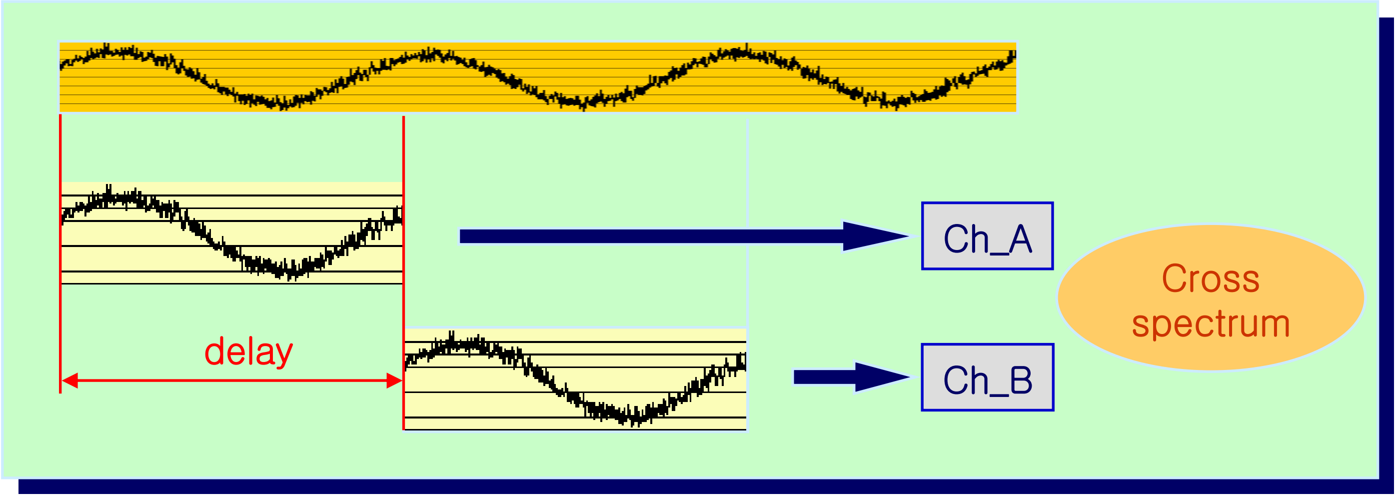
- Spectrum Enhancement (cross spectrum method)

  - Strong point :
    - Don't need Trigger signal.

  - Weak point :
    - Only random noise suppression
    - S/N improvement proportional to $\sqrt{N_A}$
    - Each signal needs dual channel analysis

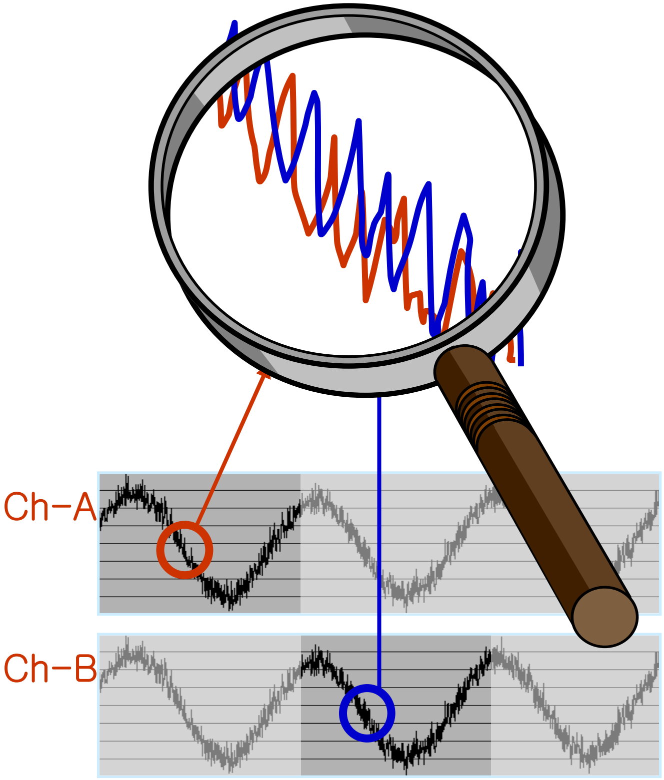


- Spectrum Enhancement (cross spectrum method)
  - Embodiment :
    - Same signal in both channels
    - Delay between channels longer than
      - record length
      - length of autocorrelation of unwanted signal
    - Display of cross spectrum



- Spectrum Enhancement (cross spectrum method)

- f(t) = Periodic signal
- m(t) = Random noise signal
- a(t), b(t) = Measured signal



- Time signal Ch_A

$$a(t) = f(t) + m(t)$$

- Spectrum Ch_A

$$A(f) = F(f) + M_1(f)$$

- Spectrum Ch_B (with time delay)

$$B(f) = F(f) \cdot e^{-j\Phi} + M_2(f)$$

- Spectrum Enhancement (cross spectrum method)
  - Cross spectrum

$$G_{AB}(f) = A^{*}(f) \cdot B(f)$$

$$= F^{*}(f) \cdot F(f) \cdot e^{-j\Phi} + M_{3}(f)$$

$$M_{3}(f) = M_{1}^{*}(f) \cdot M_{2}(f)$$

Random and Complex

  - Cross spectrum after N_A averages

$$G_{AB}(f) = G_{FF} \cdot e^{-j\Phi} + \frac{1}{\sqrt{N_{A}}} \cdot G_{M3}$$

- Spectrum Enhancement (cross spectrum method)

< Signal to Noise Ratio >

- Cross spectrum after N_A avg.

$$G_{AB}(f) = G_{FF} \cdot e^{-j\Phi} + \frac{1}{\sqrt{N_A}} \cdot G_{M3}$$

- Signal to noise improvement

$$S/N \approx \sqrt{N_A}$$

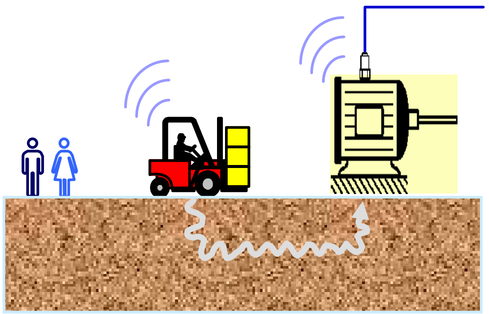
$$S/N = 10 \cdot \log \sqrt{N_A} \text{ dB}$$

$$S/N = 5 \cdot \log N_A \text{ dB}$$

- Examples

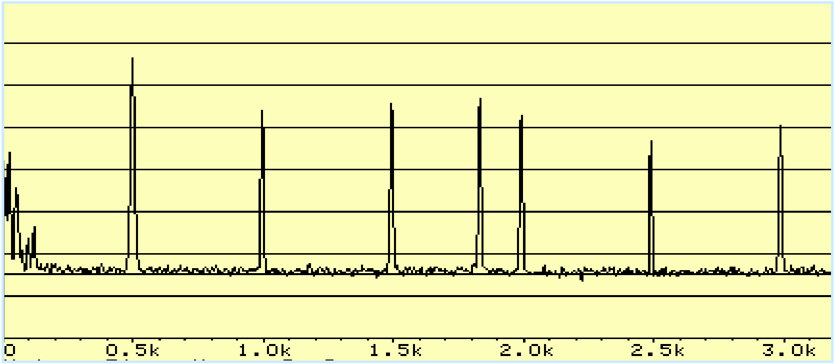
N_A	$5\log_{10}(N_A)$
10	5 dB
100	10 dB
1,000	15 dB
10,000	20 dB
32,767	23 dB

- Application of Spectrum Enhancement

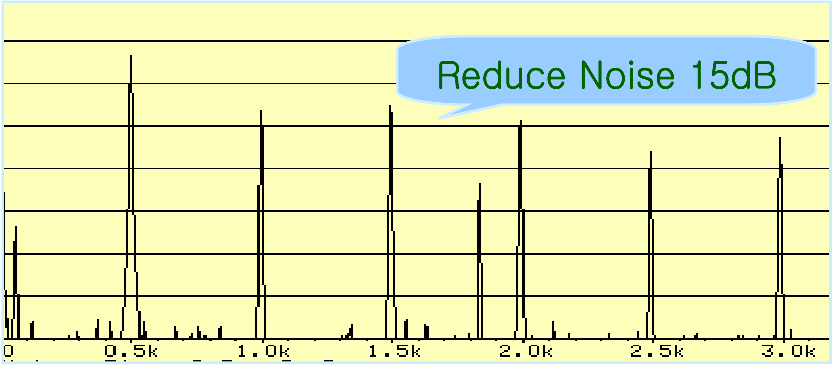


- NA = 1000 averages
- S/N improvement of 15dB

- Auto-spectrum



- Cross-spectrum



4.2 Order Analysis

- Non-Stationary Signal Analysis & Function

  - Multi Spectrum Analysis
  - Scan Analysis

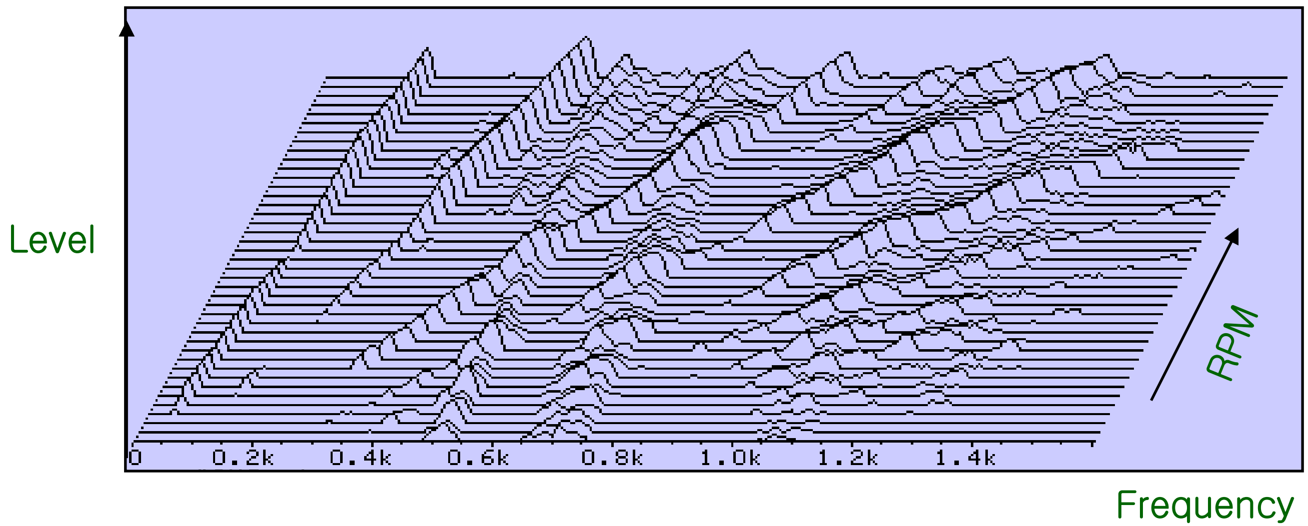
    : Divide the signal into quasi-stationary segments by proper selection of analysis window

  - STFT (Short Time Fourier Transform)

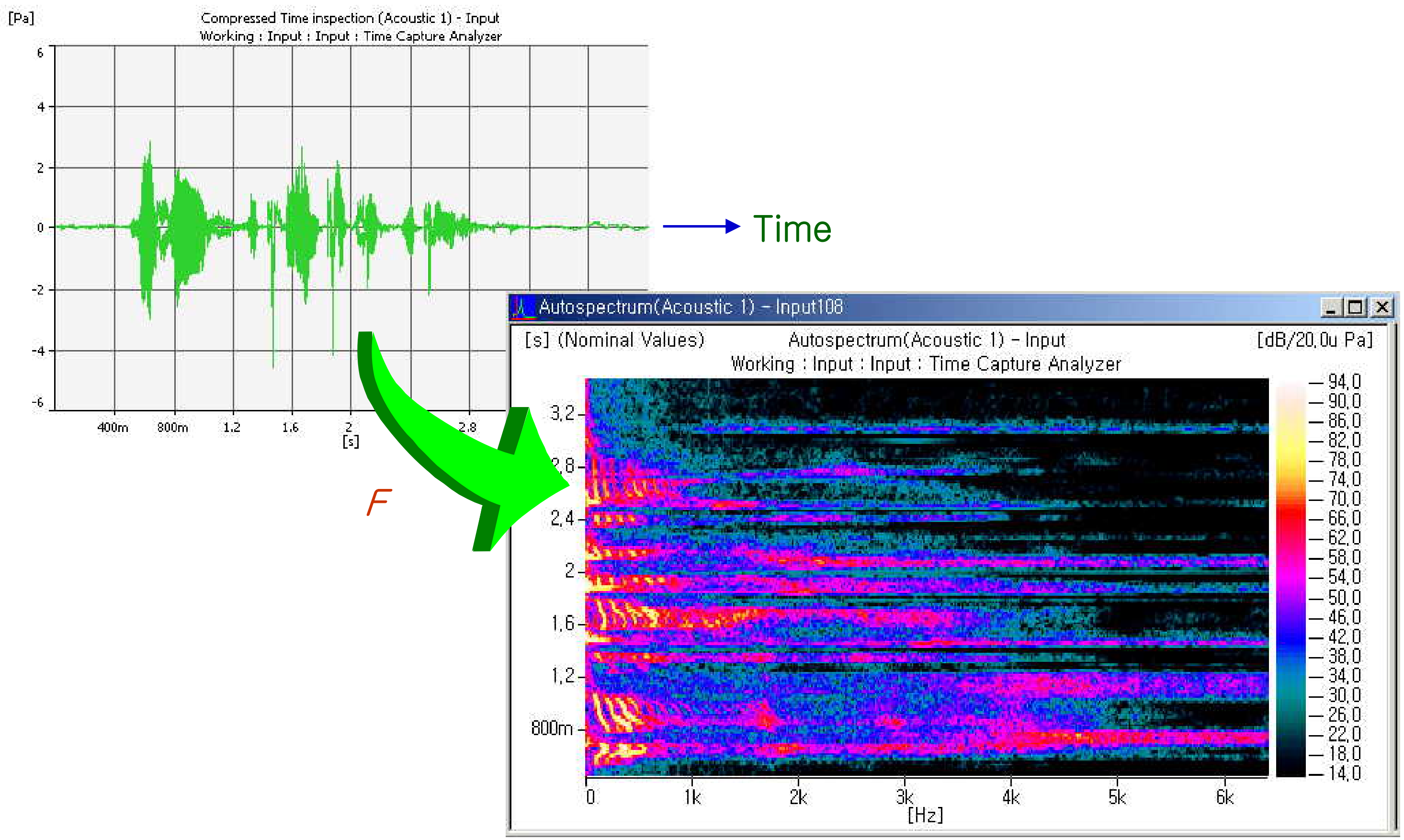
  - Order Tracking

    : Sample the signal according to its frequency variations

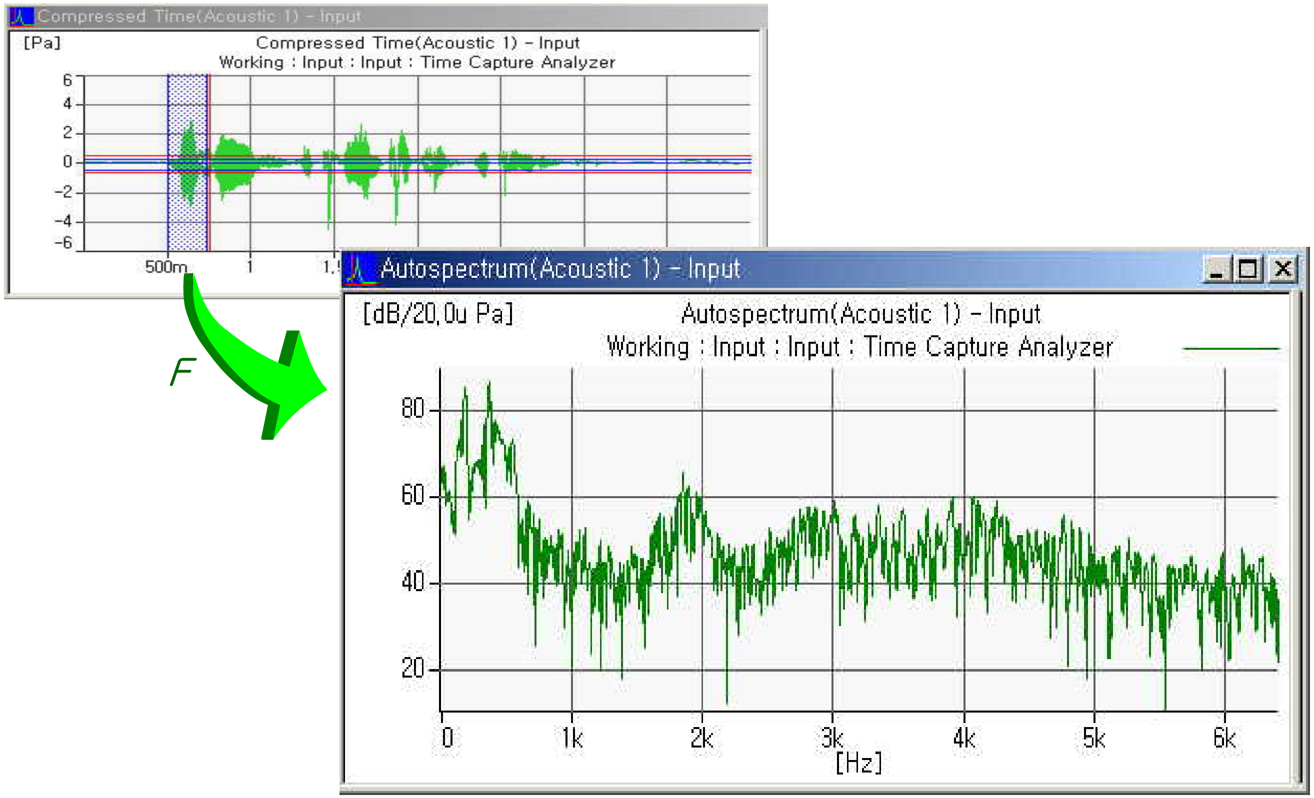
- Multi Spectrum Analysis
  - Get continuous spectrum which is prefixed
  - Using total analysis for signal
    - ex) Spectrum trend according to rpm



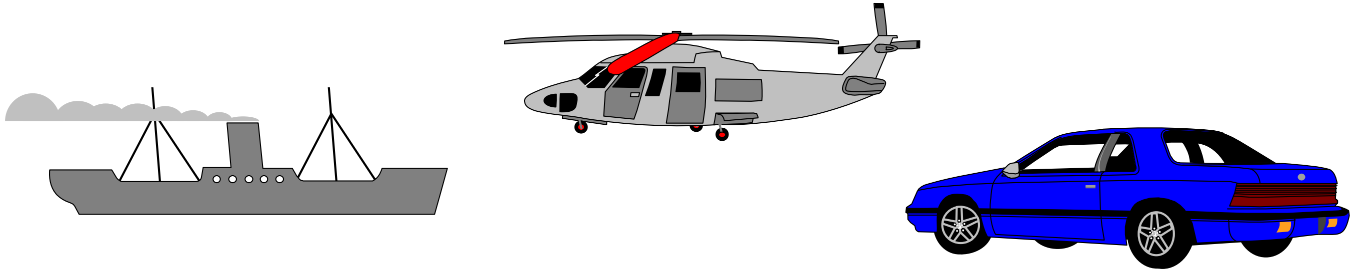
- Scan Analysis of Non-stationary Signal I



- Scan Analysis of Non-stationary Signal II



- Introduction
  - What is Order Analysis?
    - A technique for analysis of noise and vibration signals from rotating and reciprocating machinery to reveal imperfections in the moving parts
  - What is special about Order Analysis?
    - The measurement is related to the revolutions, the rotational speed, and the harmonic orders of the rotating parts

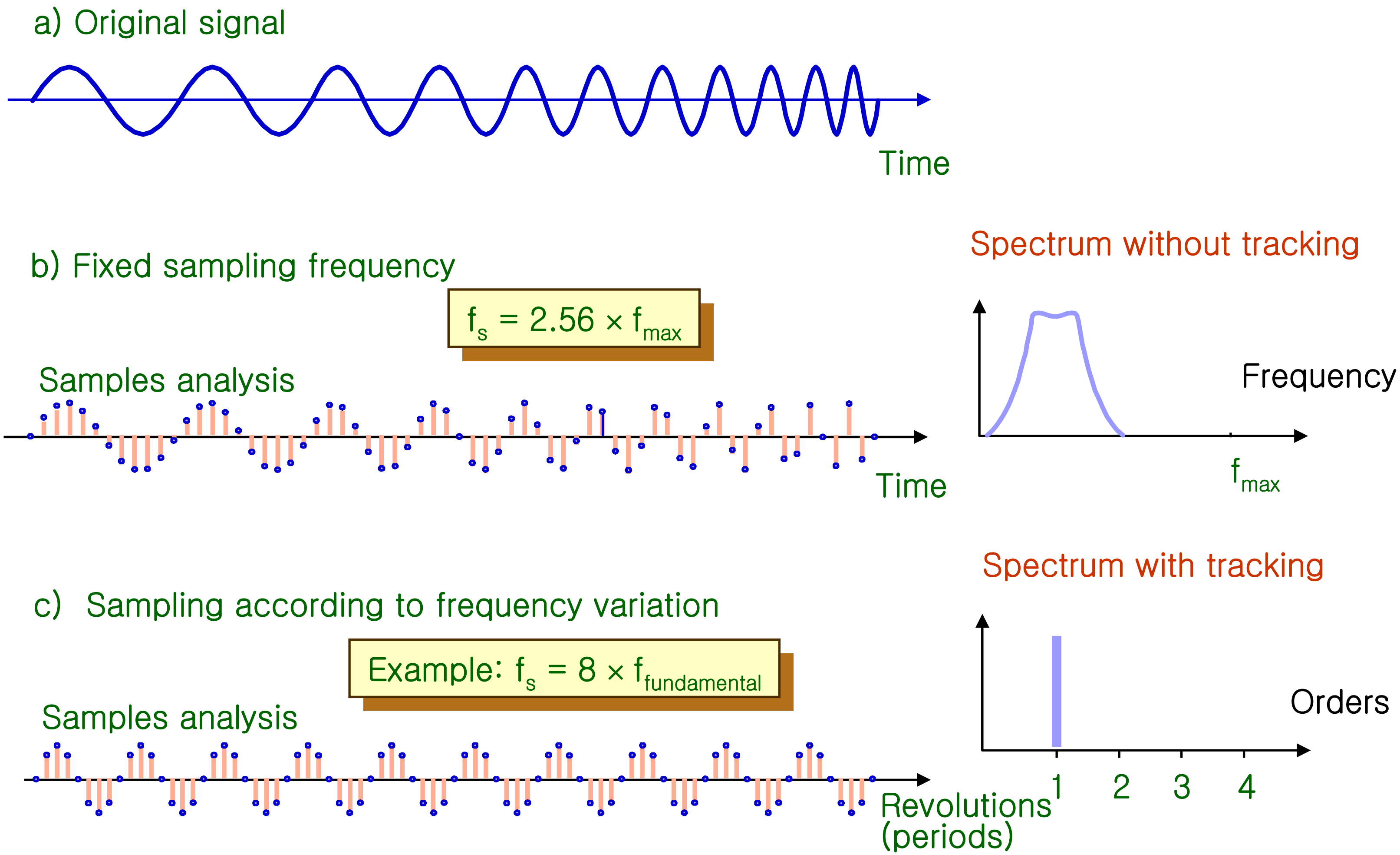


- Typical Uses for Order Analysis
  - Run-up/down
    - Separation of rotational and structural noise and vibration phenomena
    - Determination of critical speeds and resonances
    - Investigation of instabilities in rotating machinery
  - Machine Diagnostics
    - Diagnostics on machines running at almost constant speed

(Using precise trigger per revolution)

- Order Analysis – Tracking
  - Noise and vibration signals from machines with varying speed are characterized by a frequency variation of their spectral components
  - Order analysis with tracking is a method where the signal is sampled according to its frequency variation
  - The method is also called order tracking

- Analysis without and with Tracking

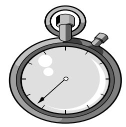


4.2 Order Analysis

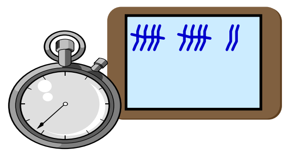
• Frequency Analysis versus Order Analysis

Frequency Analysis

Time: *When ?*
Referenced to
the clock



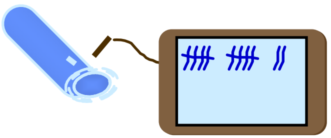
Frequency spectrum:
How often?
[per second]



Order Analysis

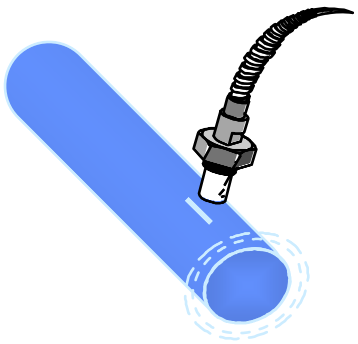
Time: *When ?*
Referenced to
the revolution
of the shaft

Order spectrum:
How often?
[per revolution]

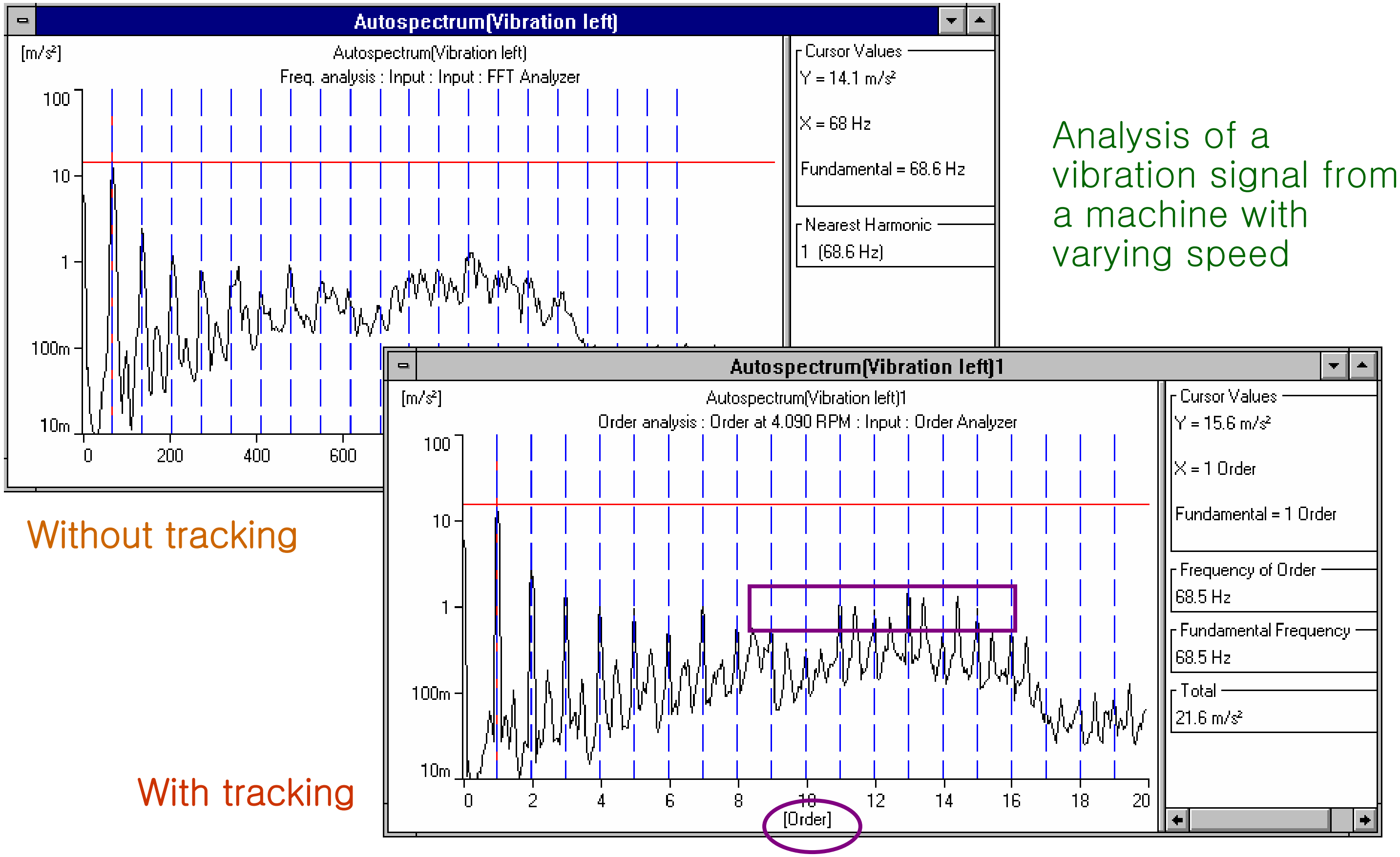


	Frequency Analysis	Order Analysis
Signal ref.	Time [sec]	Revolutions [Rev]
Spectrum	Frequency [Hz] [per sec]	Harmonics [Order] [per revolution]

- Tacho Signal
  - The key to order analysis is the measurement of the rotational speed (RPM)
  - RPM is extracted from a tacho signal
  - Tacho signal from:
    - key phasor
    - optical probe
    - shaft encoder
    - ...



- Analysis without and with Tracking



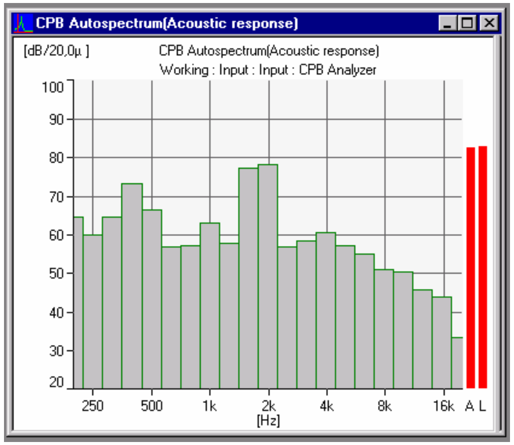
Without tracking

With tracking

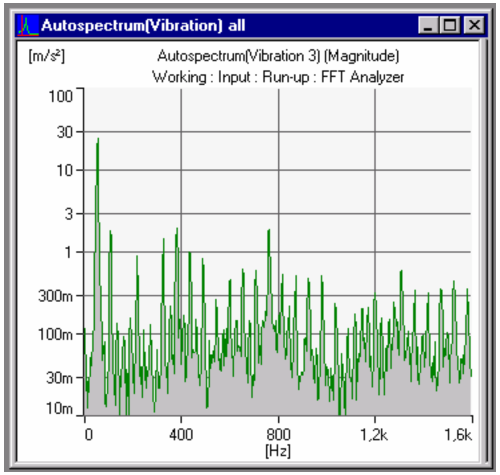
Analysis of a vibration signal from a machine with varying speed

- Overview of digital order analysis techniques

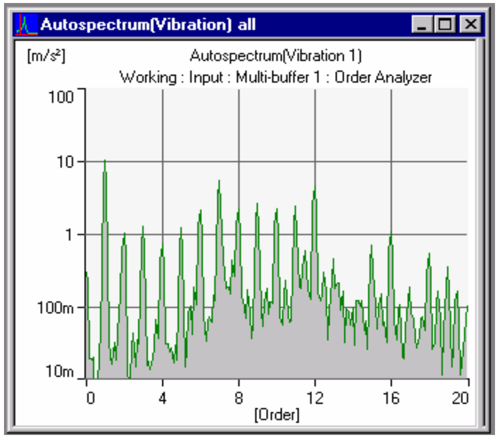
Digital Filtering based analysis including Overall levels



FFT based analysis

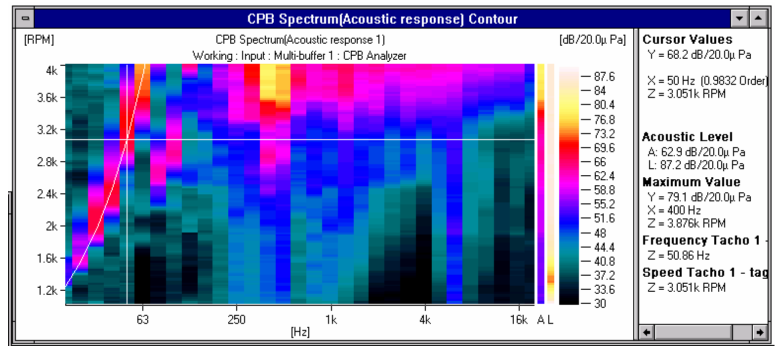


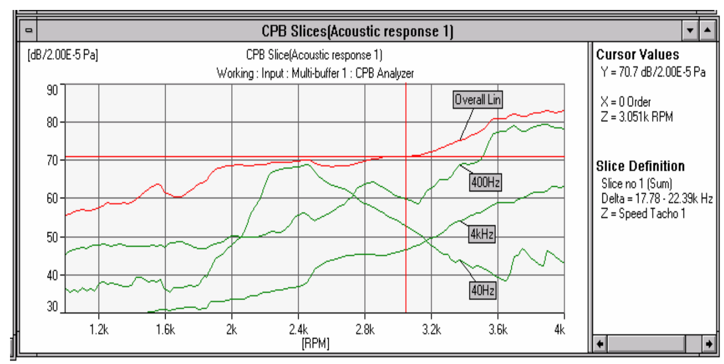
FFT/DFT based tracking analysis



• Run-up/down Test using Digital Filters

- Overall Lin, A, B, C, D levels as a function of RPM / time / other process parameters
- 1/1, 1/3, 1/12, 1/24 octave spectra as a function of RPM / time / etc.
  - low orders may be identified
  - overall and frequency slices can be extracted
- The technique can reveal frequency regions with annoying resonances
  - related to human perception of sound

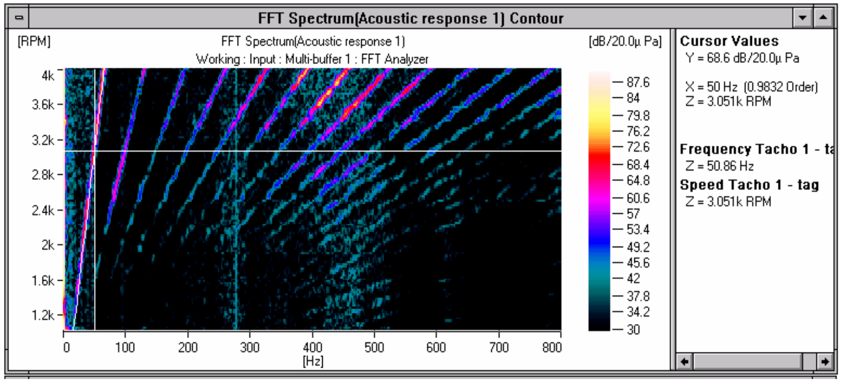


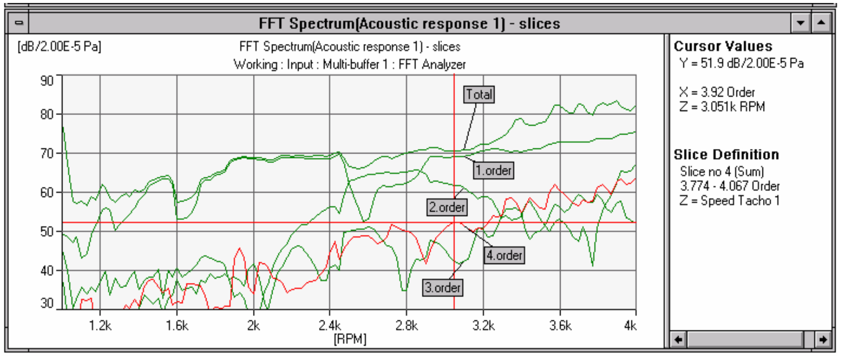


frequency slices

• Run-up/down Test using FFT Analysis

- FFT spectra as a function of RPM / time / etc.
  - fixed sampling frequency
  - fixed frequency range
- Advantages
  - fast, simple implementation
  - structural resonances parallel to RPM axis
  - order and frequency slices can be extracted
- Disadvantages
  - smearing of orders
  - limited RPM range
    - wide RPM range requires large FFT transform size

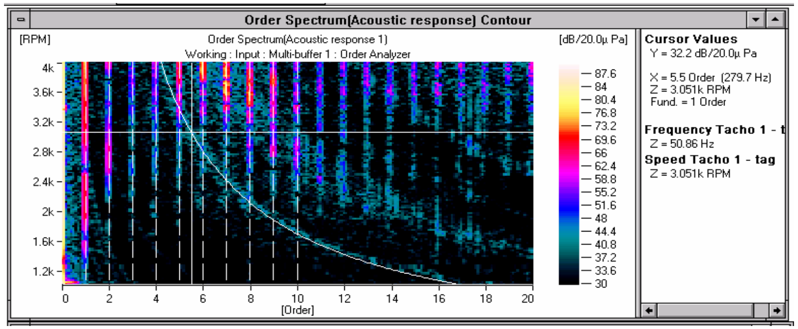




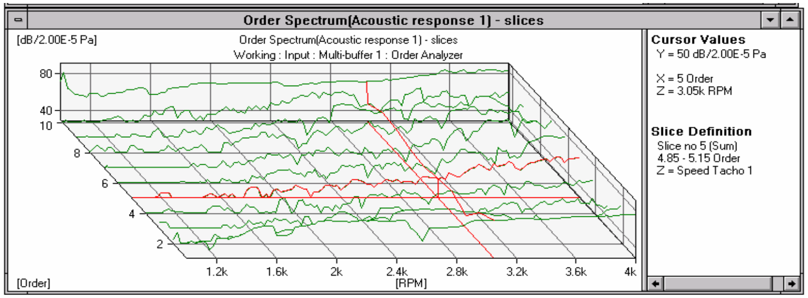
order slices

- Run-up/down test using Digital Order Tracking

  - FFT/DFT order spectra as a function of RPM / time / etc.
    - fixed order span
    - variable frequency span
    - oversampling, interpolation, resampling technique



  - Advantages
    - leakage- and smearing-free
    - identification of high orders
    - small number of lines required
    - easy order / slice extraction



order slices

  - Disadvantages
    - more complex processing
    - requires accurate tacho estimation

- Run-up/down Test without Tracking Measurement set-up considerations

Example: Analysis of the first 8 harmonic orders in a run-up from 600 RPM to 6000 RPM

a) What is the required frequency resolution ?

At lowest speed: 600 RPM ($f_{fundamental}$ = 10 Hz)

– For example 5 FFT lines per order

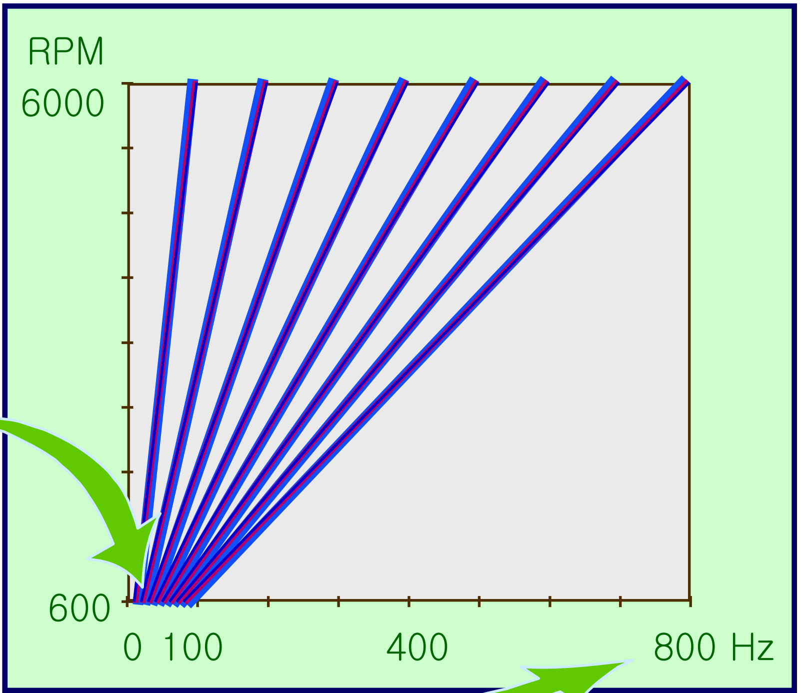
: $\triangle f$ = 2 Hz $\Rightarrow T_{record}$ = 0.5 sec

$\triangle f$ = 2 Hz

b) What is the required frequency span ?

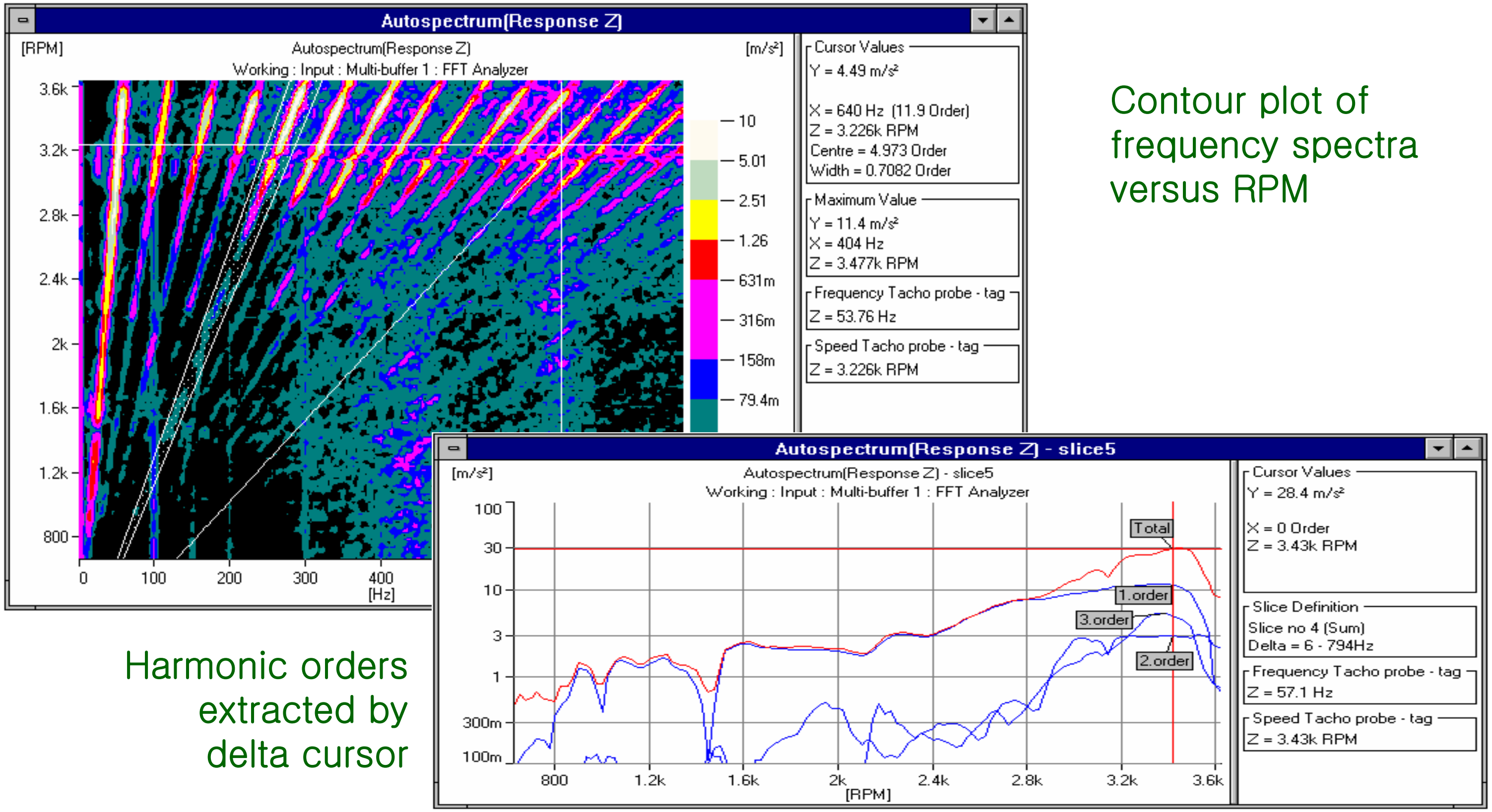
At highest speed: 6000 RPM ($f_{fundamental}$ = 100 Hz)

– 8 harmonic orders included in the analysis : Frequency Span = 800 Hz



$\triangle f$ = 2 Hz
Frequency Span = 800 Hz } 400 line frequency spectra (FFT)

- Run-up/down Tests without Tracking

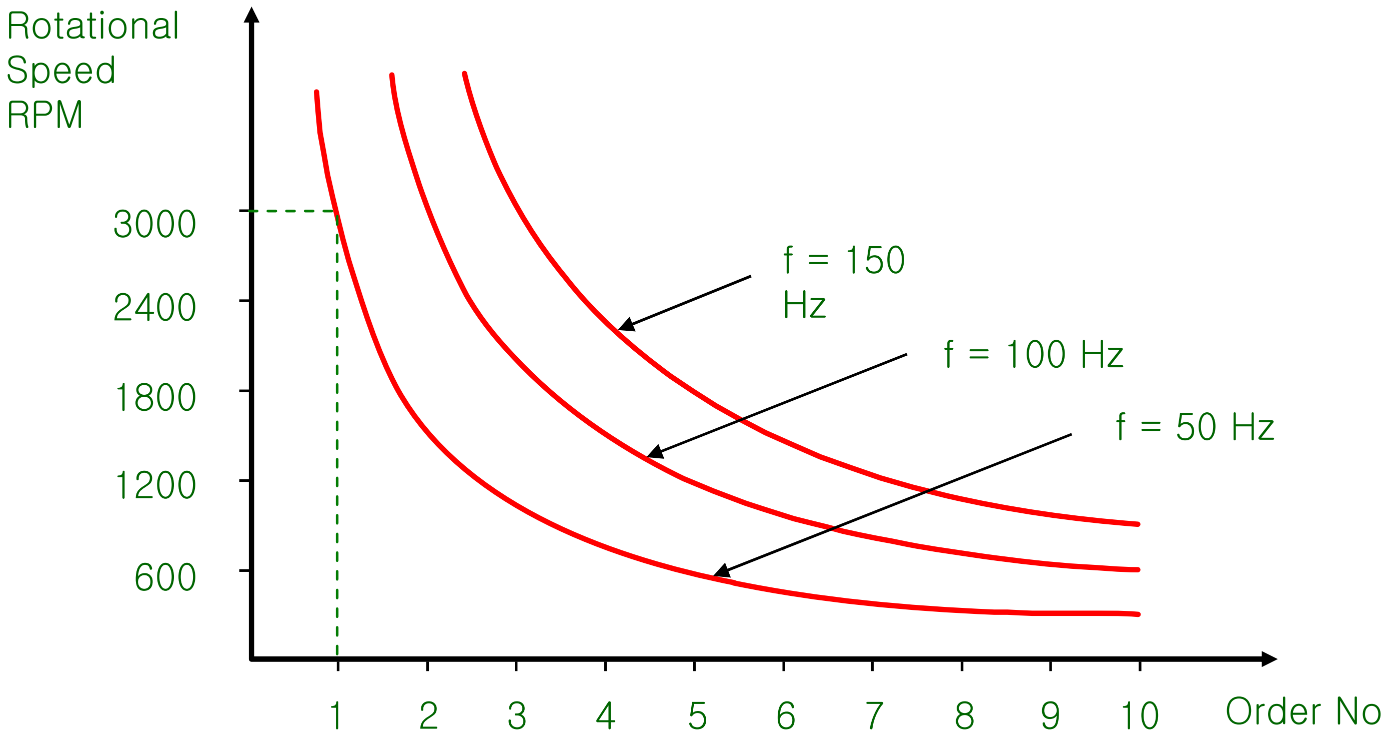


Contour plot of frequency spectra versus RPM

Harmonic orders extracted by delta cursor

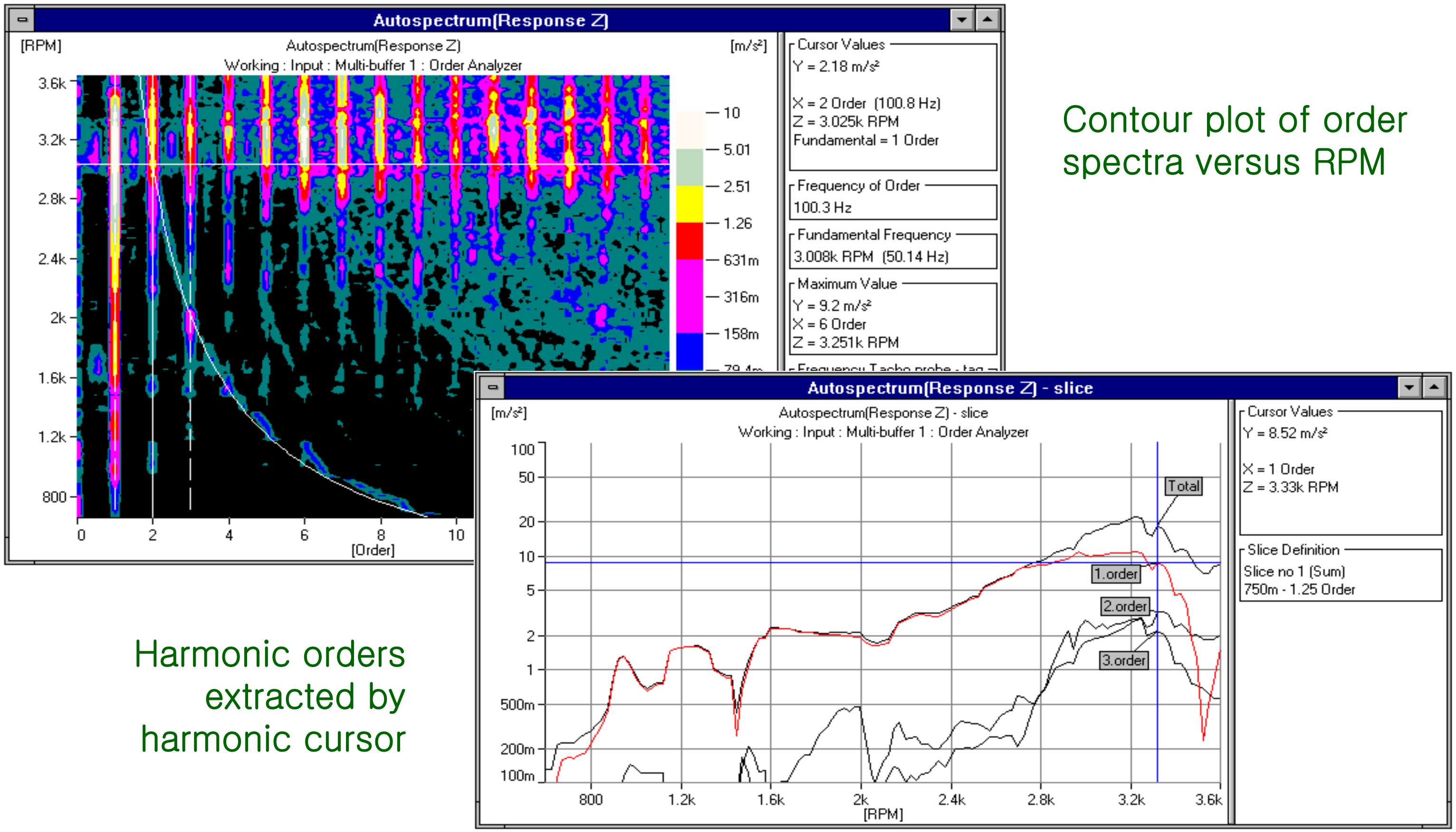
- Run-up/down Tests with Tracking

Constant Frequency Curves



4.2 Order Analysis

- Run-up/down Tests with Tracking



Contour plot of order spectra versus RPM

Harmonic orders extracted by harmonic cursor

- Conclusions Run-up/down

Run-up/down tests
without tracking:

Advantages:

- Fast
- Easy to identify resonances (fixed frequencies)

Disadvantages:

- Smearing of components
- High no. of FFT lines if wide RPM and frequency range is required

Run-up/down test
with tracking:

Advantages:

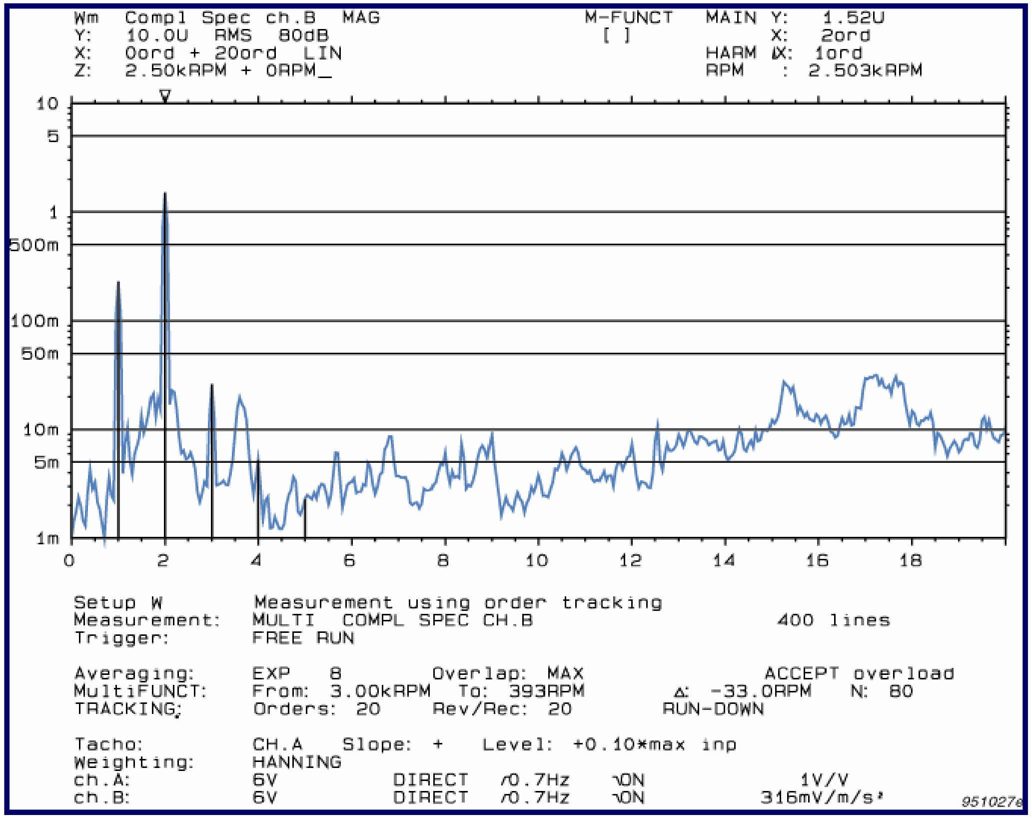
- No smearing
- Identification of higher harmonic orders

Disadvantages:

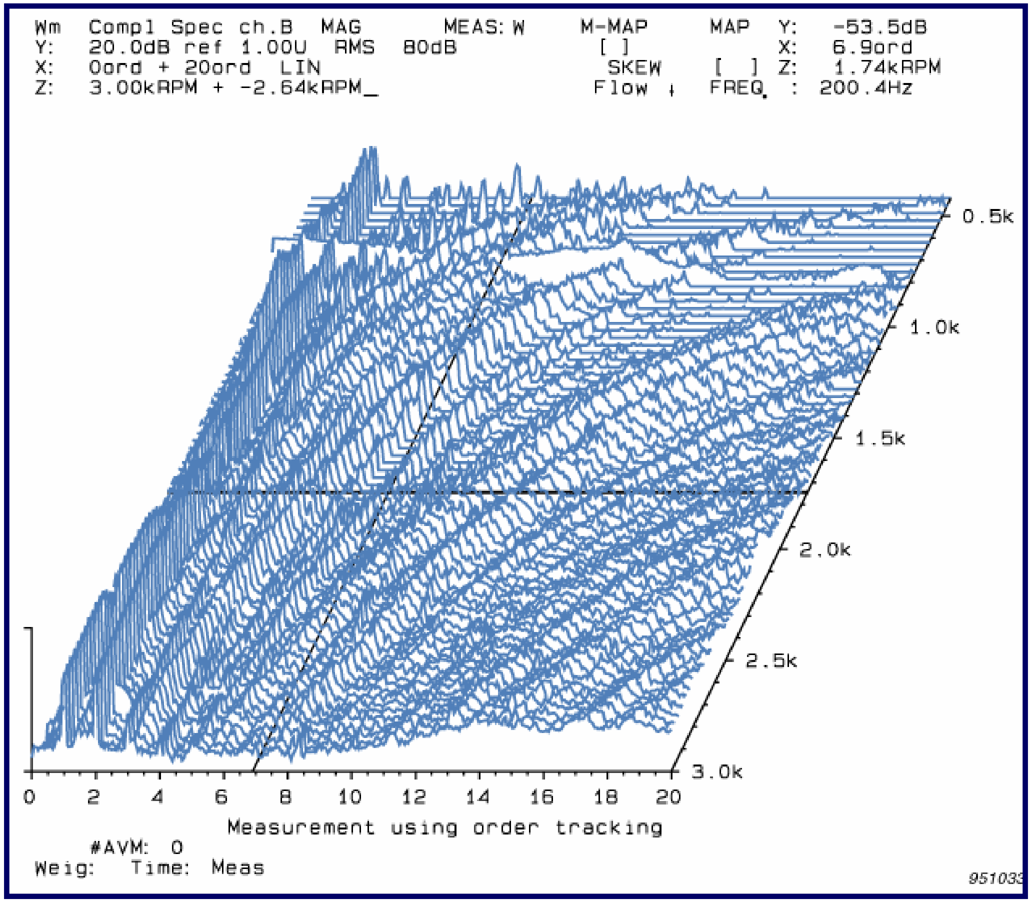
- More processing power required

- Run-down of Generator

Order spectrum at 2503 RPM



- Order Analysis of Lower Harmonics



c: Rotational speed [RPM]

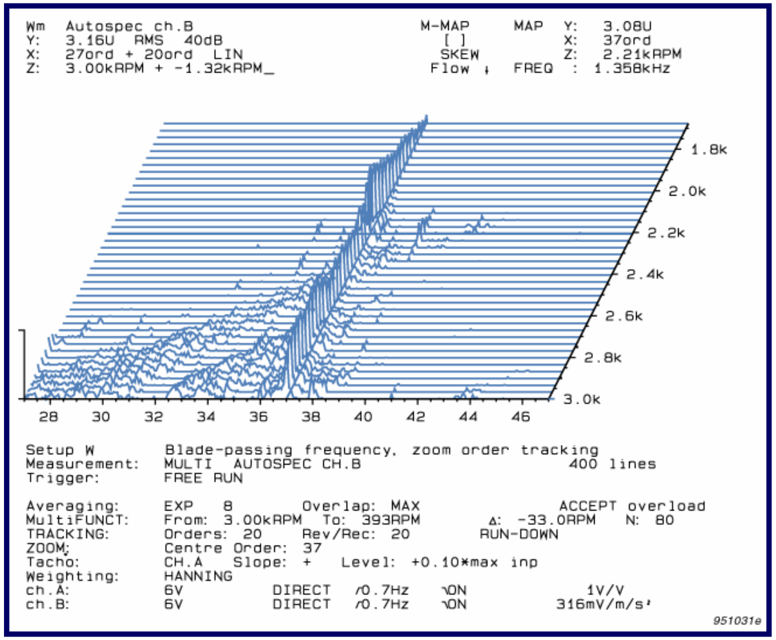
n: Order no.

f: Frequency [Hz]

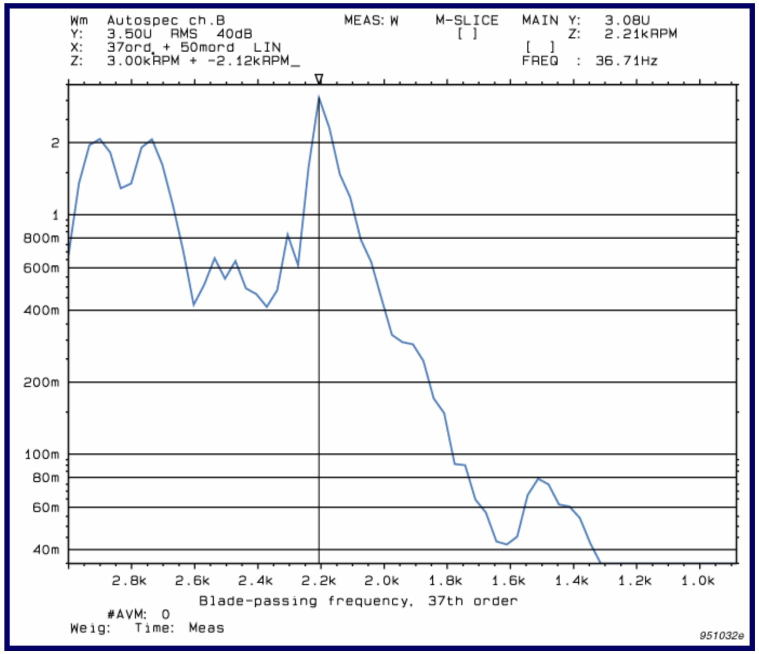
- Three significant orders
- Constant frequency lines

$$\mathbf{c \times n = f \times 60}$$

- Order Tracking diagnostics on higher harmonics

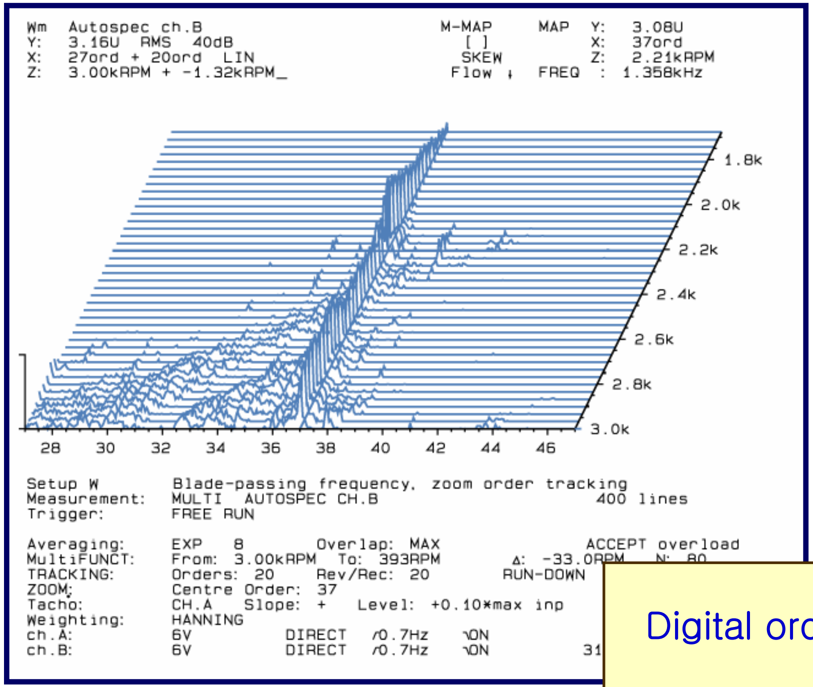


- Blade passing frequency of fan



- Identification of 37th harmonic

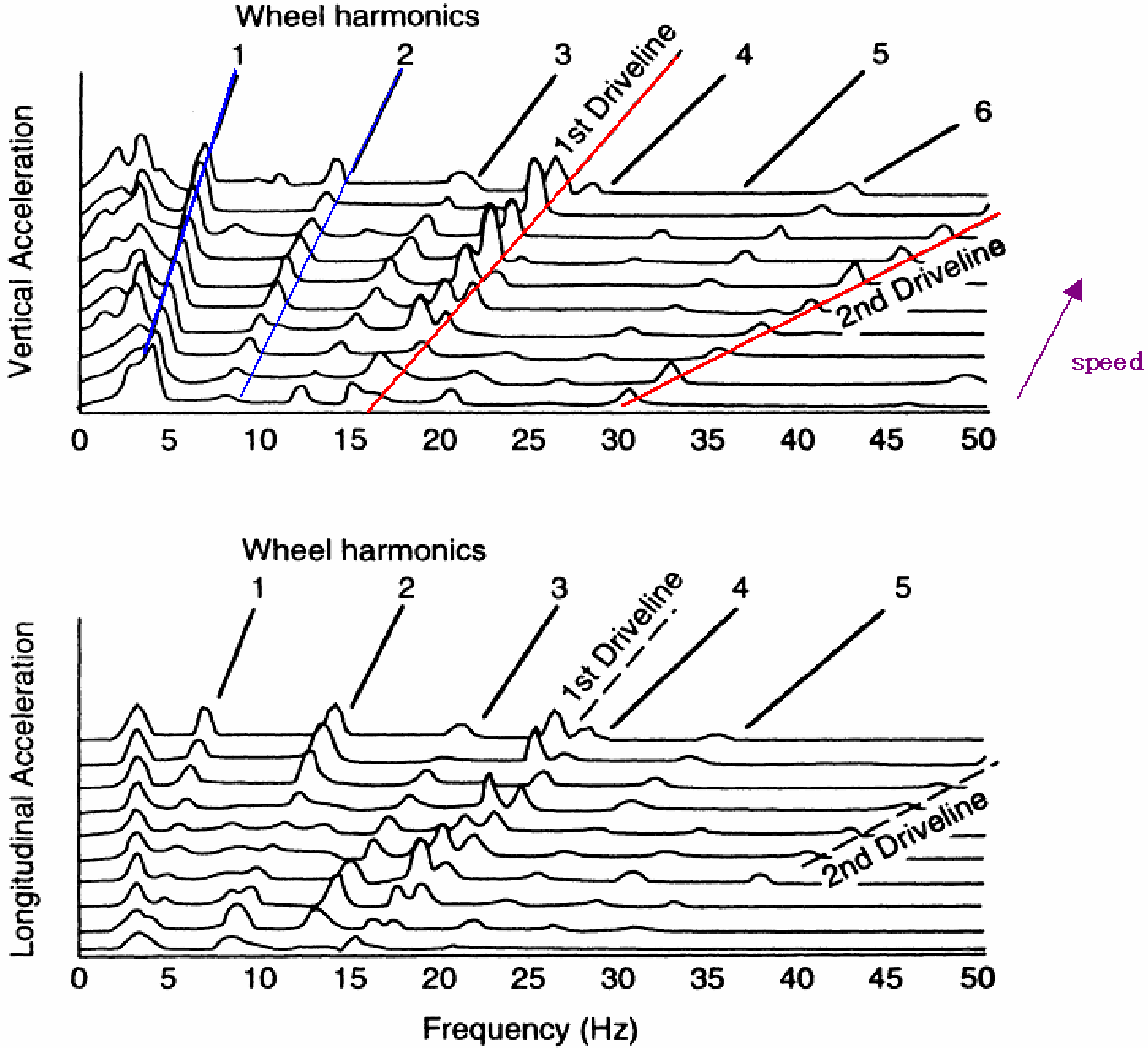
- Conclusion



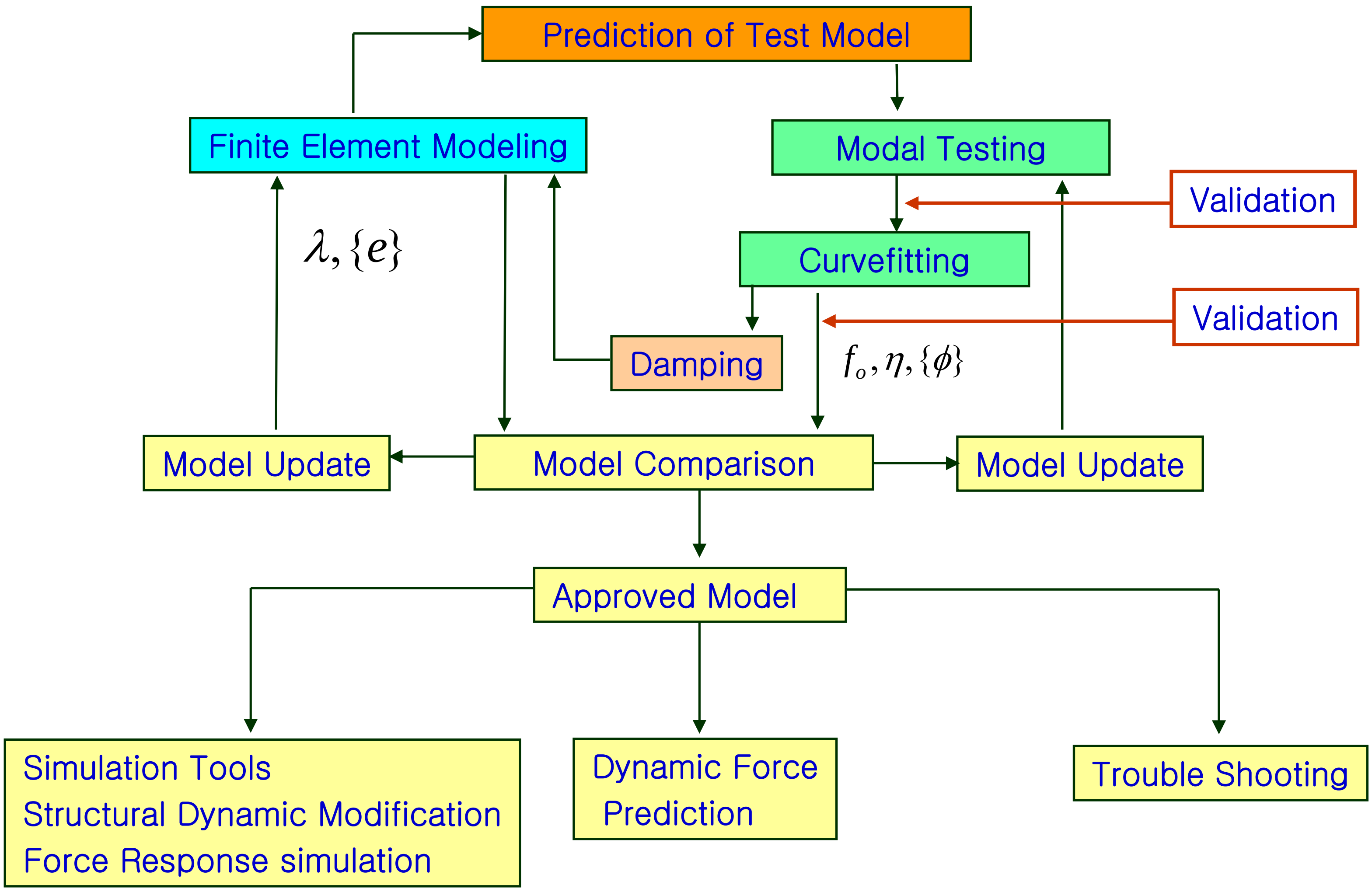
Digital order tracking identified

- Vibration caused by 37 blade fan
- Vibration NOT caused by 38 teeth gearbox

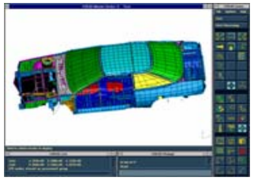
- Vehicle vibration



- System Identification

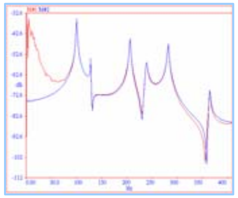


- Modal Testing

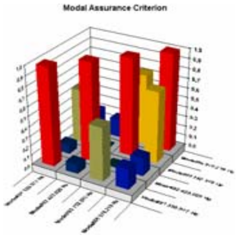


Modeling
- Geometry
- Degree of Freedom definition
- X or XYZ direction

Measurements
- Frequency Response Functions
- Hammer or shaker excitation
- Coherence Function, Autospectra etc. for validation



Curve fitting
- Frequency
- Damping
- Modeshape



Validation
- MAC (Modal Assurance Criteria)
- Synthesis
- Phase Scatter
- ....

- Purpose of lecture (curve-fitting)

  - To describe how the modal parameters are estimated from the measurements
  - To explain how the computer performs "least square fitting"
  - To discuss the use of Residual terms

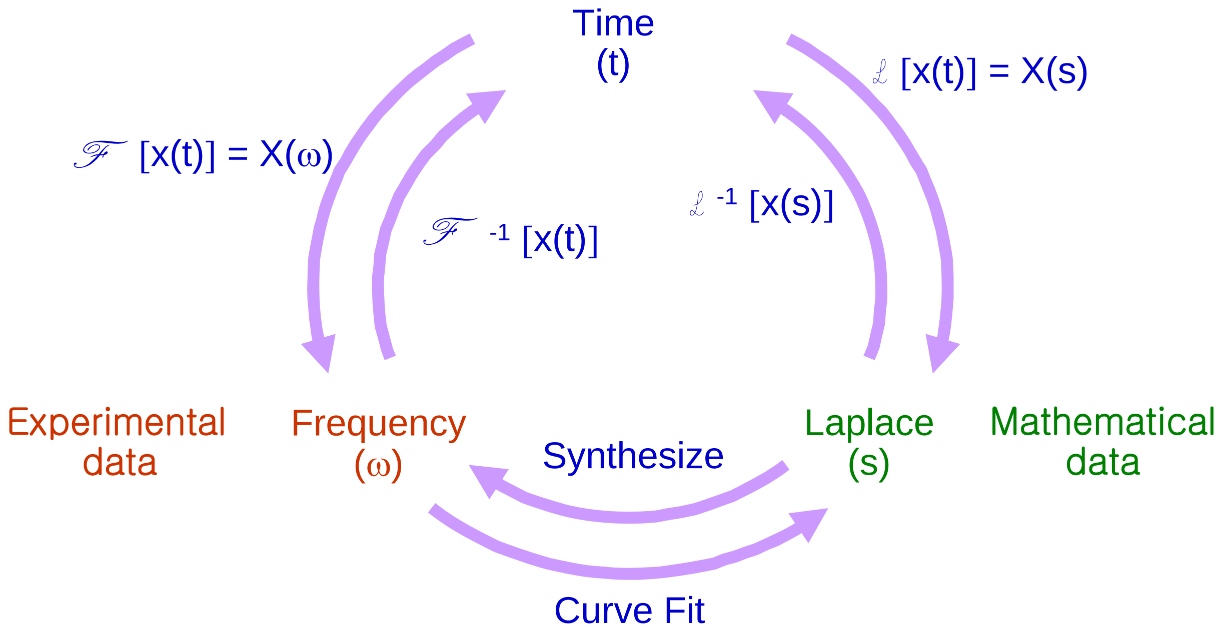
Curve Fitting:

The process of estimating the Modal Parameters from the measurements

Curve Fitting is a Data Reduction Technique

- Transformation

"Choice of domain for independent variables to simplify computations and interpretations"



- Transforms convert from one Domain into another without loss or gain of information
- Transforms are generally reversible
- Curve fitting processes are irreversible thus information may be lost
  - The lost information is hopefully noise!

- Dynamic System Identification

FORCE F(f)	×	SYSTEM H(f)	=	RESPONSE X(f)

Case	F(f)	H(f)	Y(f)	Field
1	○	○	×	Analysis
2	○	×	○	System Identification Calibration
3	×	○	○	Synthesis

- Mathematical Models: FRF

  - Undamped system

$$\alpha_{ij}(\omega) = \sum_{r=0}^{N} \frac{{}_rR_{ij}}{\omega_r^2 - \omega^2}$$

$${}_rR_{ij} = \frac{\phi_{ir}\phi_{jr}}{M_r} \quad : \quad real\ value$$

  - Proportional damping system

$$\alpha_{ij}(\omega) = \sum_{r=0}^{N} \frac{{}_rR_{ij}}{\omega_r^2 - \omega^2 + j2\varsigma_r\omega_r\omega} \Rightarrow \sum_{r=0}^{N}\left[\frac{{}_rr_{ij}}{j\omega - p_r} + \frac{{}_rr_{ij}^*}{j\omega - p_r^*}\right]$$

: General form of curve-fitting

  - Viscous damping system

$$\alpha_{ij}(\omega) = \sum_{r=0}^{N}\left[\frac{{}_rr_{ij}}{j\omega - p_r} + \frac{{}_rr_{ij}^*}{j\omega - p_r^*}\right]$$

$${}_rr_{ij} = \frac{-j\,{}_rR_{ij}}{2\omega_{dr}} = \frac{-j\phi_{ir}\phi_{jr}}{2\omega_{dr}M_r}$$

$$: \quad imaginary\ \ value$$

$$p_r = -\varsigma_r\omega_{nr} + j\sqrt{1-\varsigma_r^2}\,\omega_{nr} = -\varsigma_r\omega_{nr} + j\omega_{dr}$$

$${}_rr_{ij} = \psi_{ir}\psi_{jr} \quad : \quad complex\ value$$

- Real Mode System : Proportional damping system

$$\alpha_{ij}(\omega) = \sum_{r=1}^{n} \frac{\phi_{ir}\phi_{jr}}{K_r - \omega^2 M_r + j\omega C} = \sum_{r=0}^{N} \frac{{}_rR_{ij}}{\omega_r^2 - \omega^2 + j2\varsigma_r\omega_r\omega}$$

where $${}_rR_{ij} = \frac{\phi_{ir}\phi_{jr}}{M_r}$$

$$= \sum_{r=0}^{N} \left[\frac{{}_r r_{ij}}{j\omega - p_r} + \frac{{}_r r_{ij}^*}{j\omega - p_r^*} \right]$$

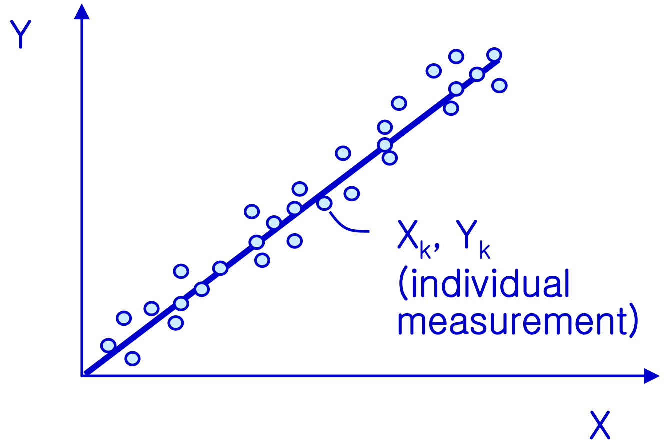
where $${}_r r_{ij} = \frac{-j\,{}_rR_{ij}}{2\omega_{dr}} = \frac{-j\phi_{ir}\phi_{jr}}{2M_r\omega_{dr}} \quad : \quad imaginary\ value$$

- Least Squares Method

Mathematical Model:

$Y_k - b \cdot X_k = N_k$

where N_k is a random error



Random Error: $N_k = Y_k - bX_k$

$$\sum N_k^2 = \sum (Y_k - bX_k)^2$$

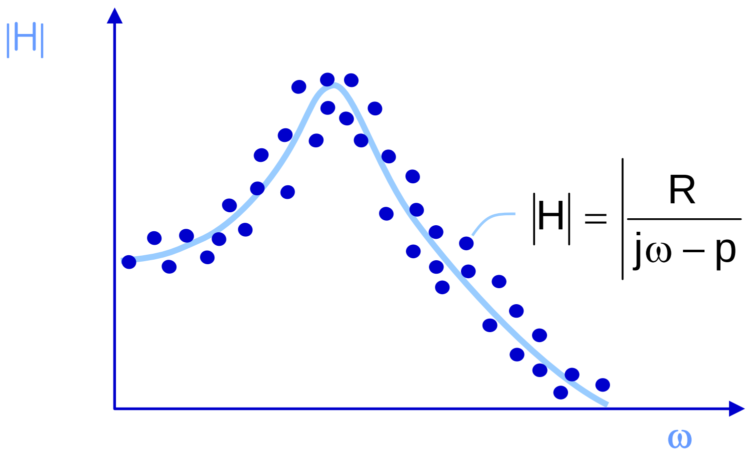
$$\frac{\partial \sum N_k^2}{\partial b} = -2\sum (Y_k - bX_k) X_k$$

$\frac{\partial \sum N_k^2}{\partial b} = 0$ gives estimate $\hat{b}$ of b which minimizes $\sum N_k^2$

$$\hat{b} = \frac{\sum Y_k X_k}{\sum X_k^2}$$

LSM – technique removes random noise

- Least Squares Method



$$|H| = \left| \frac{R}{j\omega - p} + \frac{R^*}{j\omega - p^*} \right|$$

Mathematical Model:

$$H = \frac{R}{j\omega - p} + \frac{R^*}{j\omega - p^*}$$

Method of Least Squares gives $\hat{R}, \quad \hat{p}$

- Curve Fitting Methods

Based upon measuring 1 row or 1 column of the frequency response matrix

Single Mode Methods

- Fast
- Easy to use
- Errors can occur with heavy Modal Coupling

Multiple Mode Methods

- Handle Modal Coupling
- Require more operator skill

- Curve Fitting Techniques

Single Degree of Freedom (SDOF)

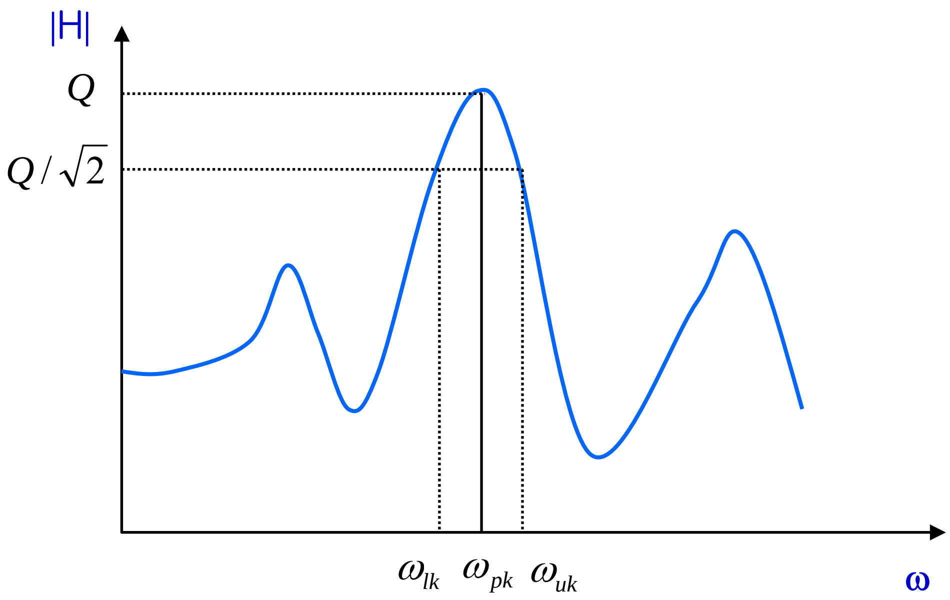
- SDOF Polynomial
- Circle Fit

Multi Degree of Freedom (MDOF)

- Complex Exponential
- Direct Parameter
- Poly-reference

- Quadratic Picking Method

  - Estimation of natural frequency and damping ratio



- k-th natural frequency : $\omega_{nk} \approx \omega_{pk}$

- k-th damping ratio : $\varsigma_k \approx \dfrac{\omega_{uk} - \omega_{lk}}{2\omega_{pk}}$

- Estimation of mode vector

$$\hat{\alpha}_j(\omega) = \begin{Bmatrix} \alpha_{1j}(\omega) \\ \alpha_{2j}(\omega) \\ \vdots \\ \alpha_{nj}(\omega) \end{Bmatrix} \quad : \text{j} - \text{th column of } \hat{\text{H}}(\omega)$$

$$\text{Im}[\hat{\alpha}_j(\omega)]_{\omega=\omega_{pk}} \propto \begin{Bmatrix} \phi_{1k} \\ \phi_{2k} \\ \vdots \\ \phi_{nk} \end{Bmatrix} \quad : \text{k} - \text{th mode shape vector}$$

This method is more precise when the mode is lightly damped and real.

- Proof)

m−th DOF

$$\alpha_{mj}(\omega) = \sum_{r=0}^{N} \left[\frac{{}_r r_{mj}}{j\omega - p_r} + \frac{{}_r r_{mj}^*}{j\omega - p_r^*} \right] \qquad p_r = -\varsigma_r \omega_{nr} + j\sqrt{1-\varsigma_r^2}\,\omega_{nr}$$

$$\frac{{}_r r_{mj}}{j\omega - p_r} + \frac{{}_r r_{mj}^*}{j\omega - p_r^*} = \frac{{}_r r_{mj}(j\omega - p_r^*) + {}_r r_{mj}^*(j\omega - p_r)}{-\omega^2 - j\omega(p_r + p_r^*) + p_r p_r^*} \qquad (\text{let } {}_r r_{mj} = a + jb)$$

$$= \frac{j\omega({}_r r_{mj} + {}_r r_{mj}^*) - ({}_r r_{mj} p_r^* + {}_r r_{mj}^* p_r)}{-\omega^2 + j2\varsigma\omega_r\omega + \omega_r^2} = \frac{2j\omega a - 2(-\varsigma\omega_r a + \sqrt{1-\varsigma^2}\,\omega_r b)}{\omega_r^2 - \omega^2 + j2\varsigma\omega_r\omega}$$

$$\text{If} \quad {}_r r_{mj} = \psi_{mr}\psi_{jr}, \quad \omega = \omega_r, \quad \varsigma \cong 0 \quad \Rightarrow \quad a \approx 0$$

$$\alpha_{mj}(\omega) = \frac{2j\omega_r a - 2\omega_r b}{j2\varsigma\omega_r\omega_r} = \frac{a + jb}{\varsigma\omega_r} = \frac{\psi_{mr}\psi_{jr}}{\varsigma\omega_r} \qquad \mathrm{Im}[\hat{\alpha}_j(\omega)]_{\omega=\omega_{pk}} = \begin{Bmatrix} \dfrac{\psi_{1r}\psi_{jr}}{\varsigma\omega_r} \\ \dfrac{\psi_{2r}\psi_{jr}}{\varsigma\omega_r} \\ \vdots \\ \dfrac{\psi_{nr}\psi_{jr}}{\varsigma\omega_r} \end{Bmatrix}$$

- SDOF Polynomial Curve Fit

$$H(\omega) = \left(\frac{R}{j\omega - p} + \frac{R^*}{j\omega - p^*} \right) + A_0 + A_1\omega + A_2\omega^2$$

$R = R_1 + jR_2$ is the residue

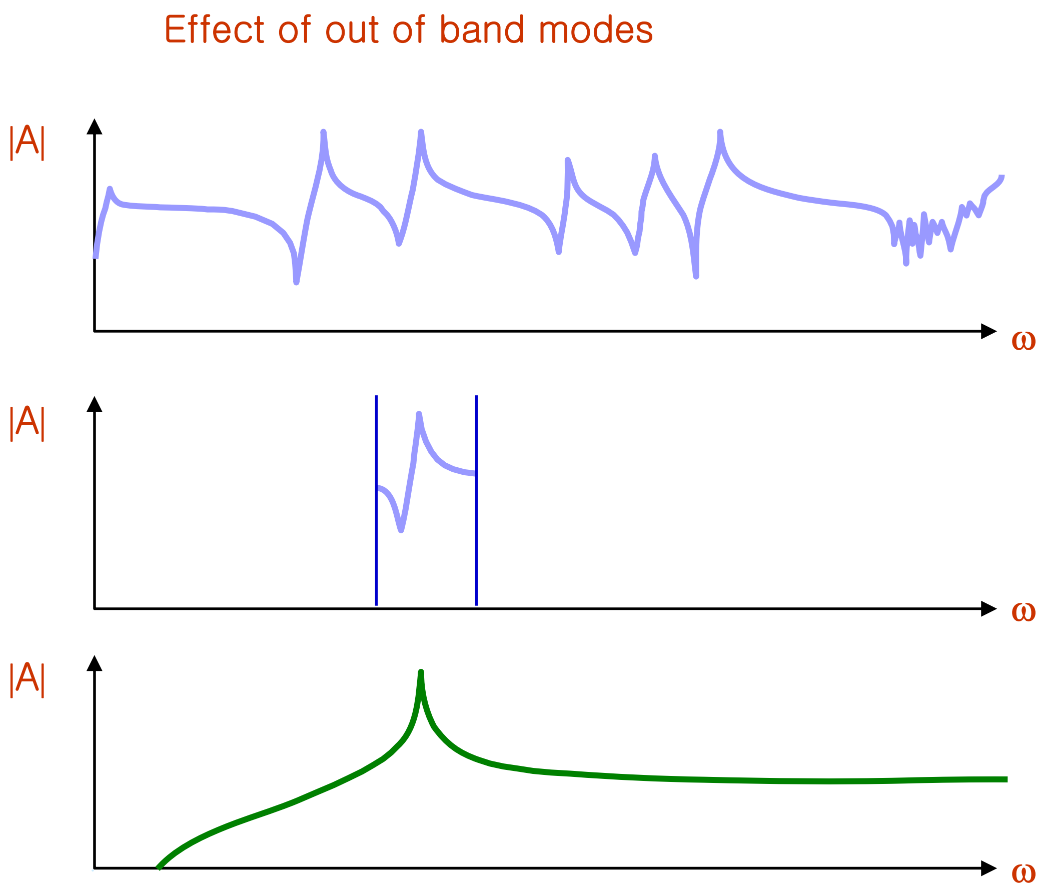
$p = \sigma + j\omega_d$

σ is the Decay Rate

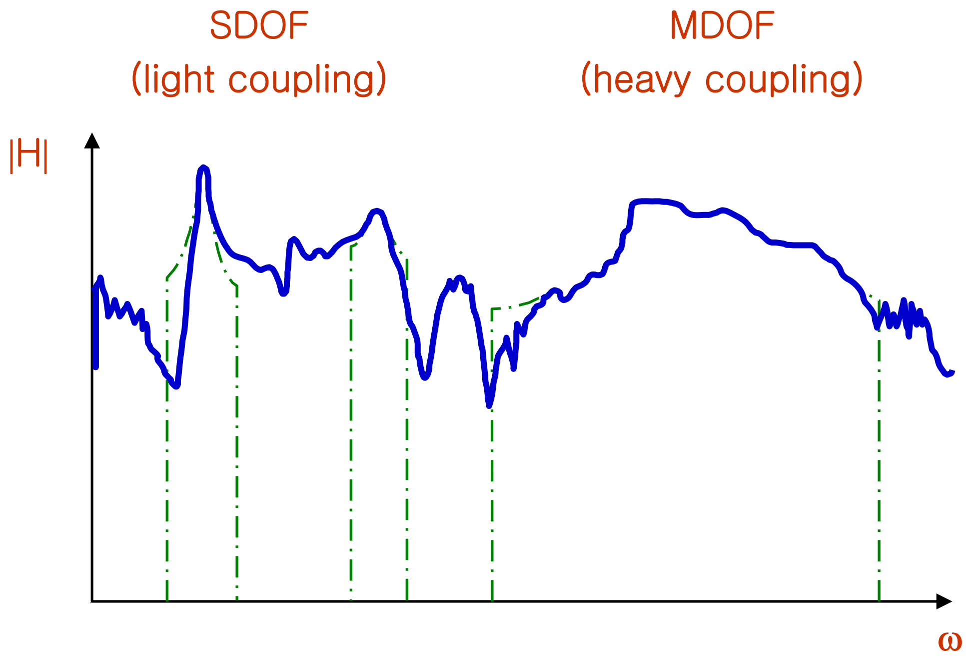
ω_d is the Damped Natural Frequency

A_0, A_1, A_2 are Residual function constants (Effect of out-of-band modes)

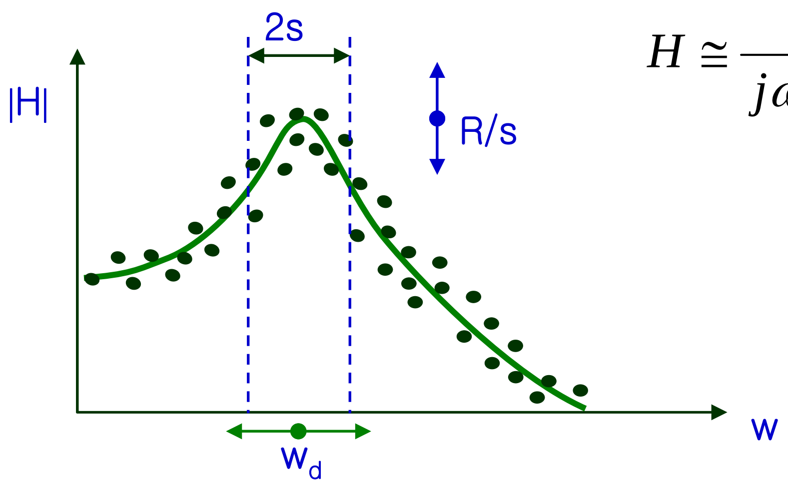
- SDOF Polynomial Curve Fit



- Combining SDOF and MDOF Methods
  - use SDOF methods on LIGHTLY COUPLED modes
  - use MDOF methods on HEAVILY COUPLED modes



- Summary
  - Curve Fitting is the process of estimating the Modal Parameters from the measurements



$$H \cong \frac{R}{j\omega - (-\sigma + j\omega_d)} + \frac{R^*}{j\omega - (-\sigma - j\omega_d)}$$

where

$$\sigma = \omega_o \cdot \zeta$$

$$\omega_d = \omega_o \sqrt{1 - \zeta^2}$$

Curve Fitting is a *data reduction* technique

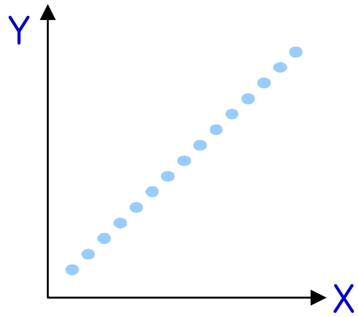
- Thus information may be lost
- The lost information is hopefully noise!

- Modal Testing Validation Tools

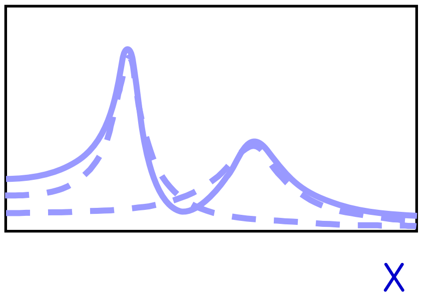
  - Testing
    - Coherence
    - Autospectrum at Driving Point $H(f,F_1) = H(f,F_2)$
    - Reciprocity $H_{ij}(f) = H_{ji}(f)$
    - Driving Point
    - Linearity

  - Curve-fitting
    - Synthesis
    - Modal Assurance Criteria
    - Phase Scatter

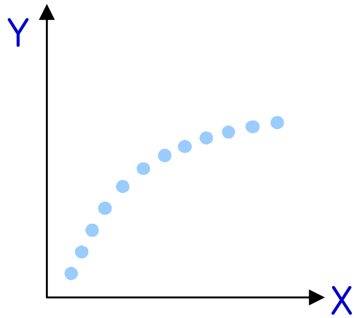
- Linearity



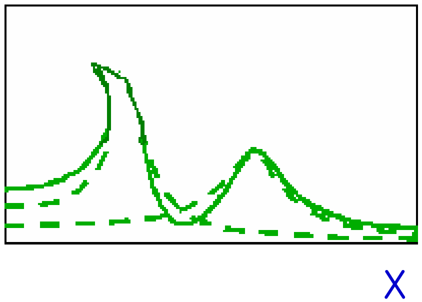
Linear



$$Y(\omega) = H(\omega)X(\omega)$$



Nonlinear



- The Coherence Function

  - Input-Output Relationship

$$[\text{Cross Power}]^2 \le [\text{Input Power}] \cdot [\text{Output Power}]$$

$$|G_{AB}(f)|^2 \le |G_{AA}(f)| \cdot |G_{BB}(f)|$$

  - Definition

$$A(t) = a(t) + m(t)$$

$$B(t) = b(t) + n(t)$$

$$\gamma^2(f) \equiv \frac{|G_{AB}(f)|^2}{|G_{AA}(f)| \cdot |G_{BB}(f)|}$$

$$\gamma^2(f) \equiv \frac{|G_{ab}(f)|^2}{|G_{aa}(f) + G_{mm}(f)| \cdot |G_{bb}(f) + G_{nn}(f)|}$$

  - It expresses degree of *linear* relationship between A(f) and B(f)

$$0 \le \gamma^2(f) \le 1$$

- Proof)

$$G_{xy}(f) = |G_{xy}(f)| e^{-j\theta_{xy}(f)} \qquad G_{yx}(f) = G_{xy}^{*}(f) = |G_{xy}(f)| e^{j\theta_{xy}(f)} \qquad \theta_{xy} = \theta_x - \theta_y$$

Consider now the quantities $X(f)$ and $Y(f)e^{j\theta_{xy}}$

For any real constants a, b : → a/b : real value

$$(aX + bYe^{j\theta_{xy}})(aX^{*} + bY^{*}e^{-j\theta_{xy}}) \geq 0$$

$$a^2X^{*}X + ab(X^{*}Ye^{j\theta_{xy}} + XY^{*}e^{-j\theta_{xy}}) + b^2Y^{*}Y \geq 0$$

$$a^2G_{xx} + ab\left(G_{xy}e^{j\theta_{xy}} + G_{yx}e^{-j\theta_{xy}}\right) + b^2G_{yy} \geq 0$$

$$a^2G_{xx} + 2ab|G_{xy}| + b^2G_{yy} \geq 0$$

$$G_{xx}\left(\frac{a}{b}\right)^2 + 2|G_{xy}|\left(\frac{a}{b}\right) + G_{yy} \geq 0 \quad \Longrightarrow \quad |G_{xy}|^2 - G_{xx}G_{yy} \leq 0$$

- Coherence vs Correlation Coefficient

  - Coherence

$$\gamma^2(f) \equiv \frac{\left|G_{AB}(f)\right|^2}{G_{AA}(f) \cdot G_{BB}(f)} \qquad 0 \le \gamma^2(f) \le 1$$

  - Correlation Coefficient

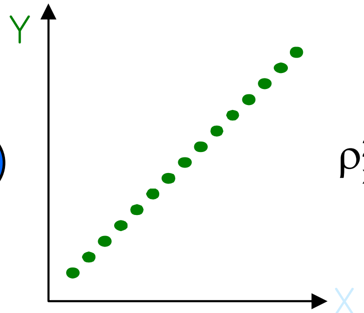
$$\rho_{xy}^2 \equiv \frac{\sigma_{xy}^2}{\sigma_x^2 \cdot \sigma_y^2} \qquad 0 \le \rho_{xy}^2 \le 1$$

Variance ~ Autospectrum

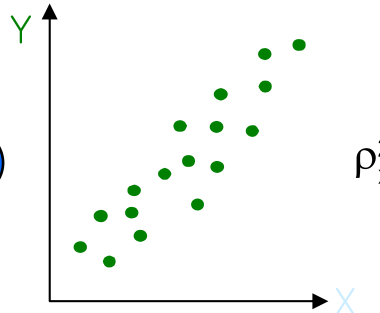
Covariance ~ Cross Spectrum

- The Correlation Coefficient

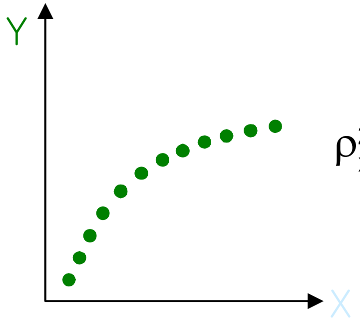
$$\rho_{xy}^2 = \frac{\sigma_{xy}^2}{\sigma_x^2 \cdot \sigma_y^2}$$

1 

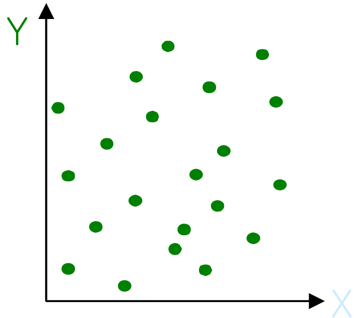
$\rho_{xy}^2 = 1$

2 

$\rho_{xy}^2 < 1$

3 

$\rho_{xy}^2 < 1$

4 

$\rho_{xy}^2 = 0$

- The Coherence Function

$$\gamma^2(f) \equiv \frac{\left|G_{AB}(f)\right|^2}{G_{AA}(f) \cdot G_{BB}(f)}$$

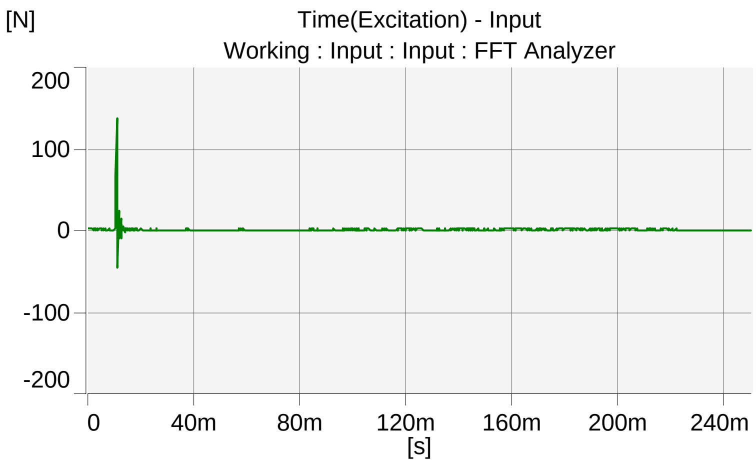
Expresses the degree of linearity

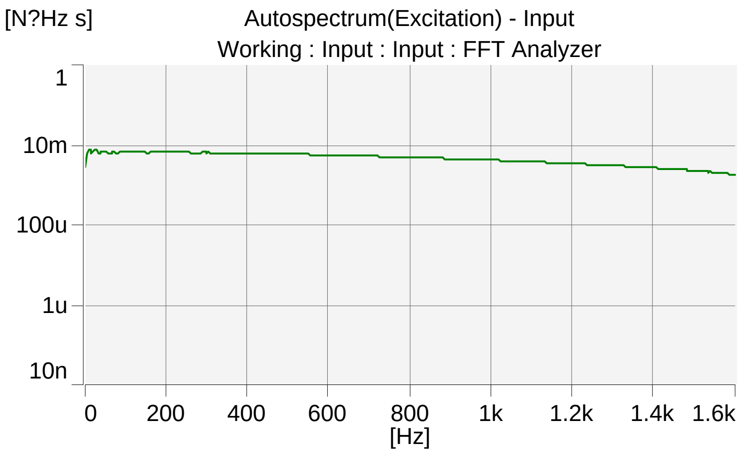
The coherence only provides useful information when $G_{AB}(f)$, $G_{AA}(f)$ and $G_{BB}(f)$ are averaged over many records

- Difficult measurements:
  - Noise in measured output signal
  - Noise in measured input signal
  - Other inputs not correlated with measured input signal

- Bad measurements:
  - Leakage
  - Time varying systems
  - Non-linearities of system
  - DOF-jitter
  - Propagation time not compensated for

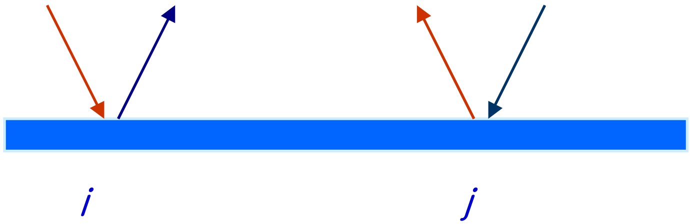
- Autospectrum at Driving Point
  - Ensure the desired frequency range is excited





- Reciprocity

$$H_{ij} = H_{ji}$$



Validation of

- Linearity
- Homogenous
- Time invariance

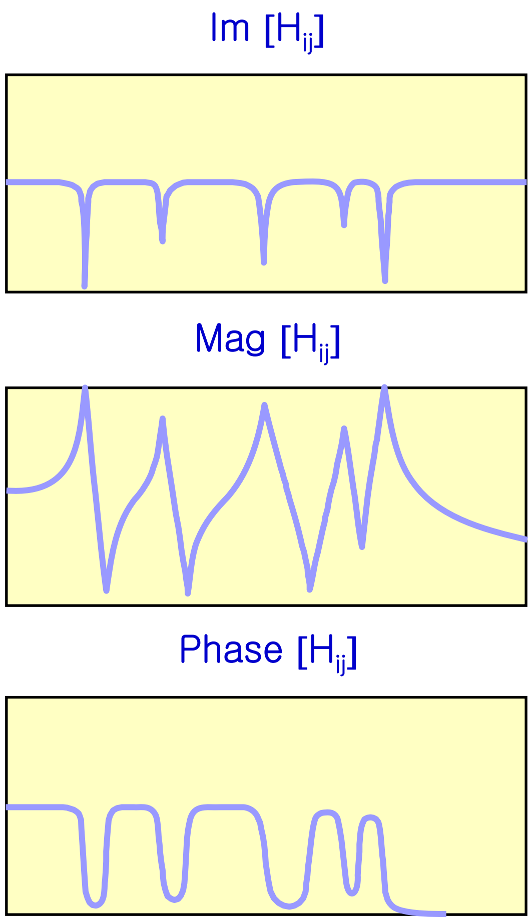
- Check of Driving Point Measurement

  - All peaks in

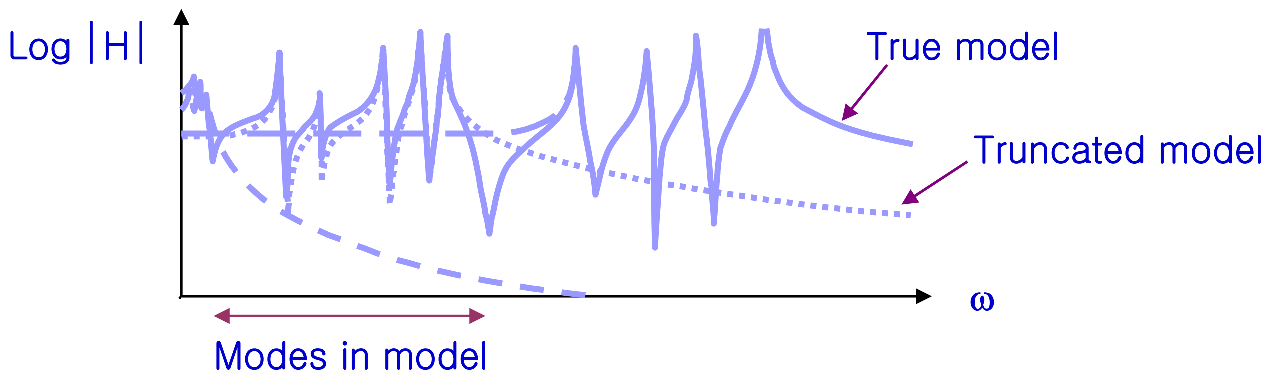
    $$\mathrm{Im}\left[\frac{X(f)}{F(f)}\right],\ \mathrm{Re}\left[\frac{\dot{X}(f)}{F(f)}\right]\ \text{and Im}\left[\frac{\ddot{X}(f)}{F(f)}\right]$$

  - An anti-resonance in Mag $[H_{ij}]$ must be found between every pair of resonances

  - Phase fluctuations must be within 180°



- Synthesis
  - Truncated Models (Modes)



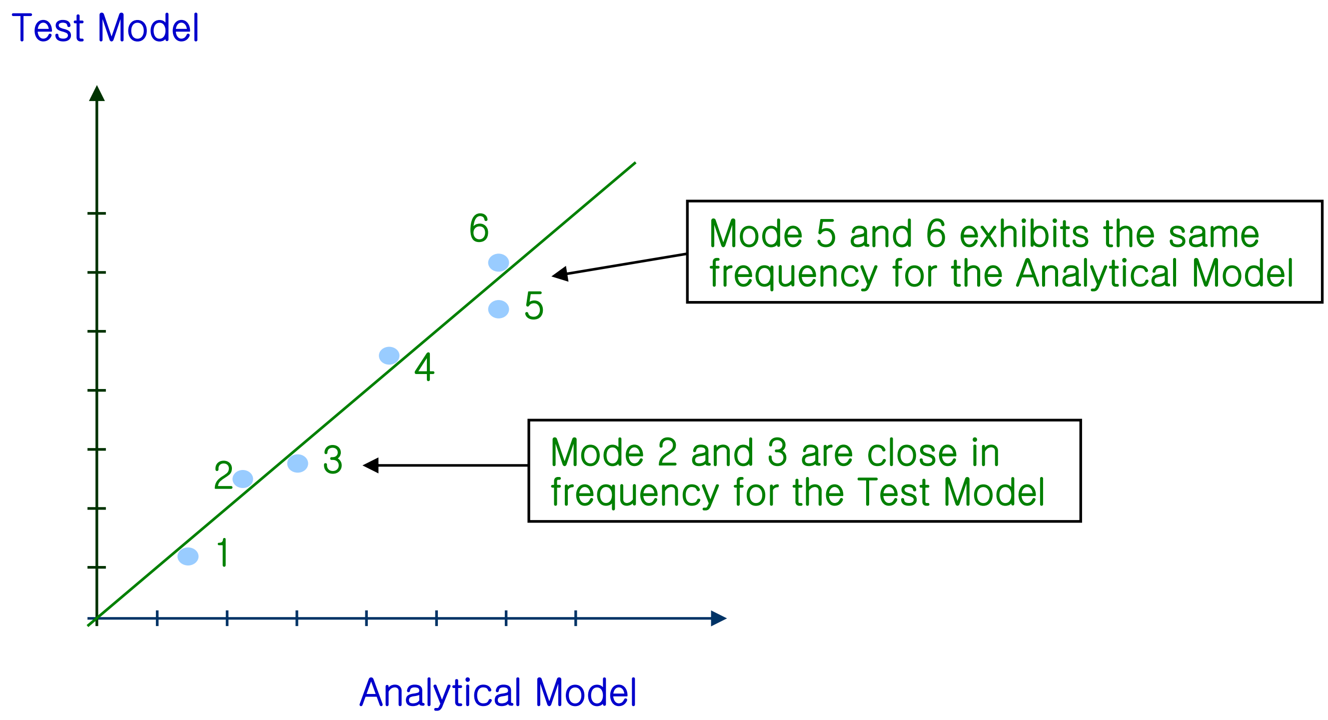
Truncated Model: $$H_{ij} = \sum_r \frac{R_{ijr}}{j\omega - p_r} + \frac{R^*_{ijr}}{j\omega - p^*_r}$$

"Improved" Model: $$H'_{ij} = \frac{1}{m_{res}\omega^2} + H_{ij} + \frac{1}{k_{res}}$$

Residual terms

- The residual terms compensate for the out of band modes, but cannot be used in the synthesis of non-measured FRF's
- The Modal Model cannot be improved by adding residual terms
- Residual terms are used in curve fitting

- Modal Comparison
  - Comparison of natural frequencies
    - Sorting the Modes by frequency
    - Tabular versus graphic results



- Modal Assurance Criteria (MAC)

The MAC function is a scalar constant relating two modeshapes

- From different models (as FE Model vs. Test model)
- From same model (Validation tool often called Auto MAC)

MAC function correlation of the two modeshapes

$$\{\phi_x\} \quad and \quad \{\phi_a\}$$

$$MAC(a,x) = \frac{\left|\{\phi_x\}^T \{\phi_a\}\right|^2}{\left(\{\phi_x\}^T \{\phi_x\}\right)\left(\{\phi_a\}^T \{\phi_a\}\right)}$$

Orthogonality

$$\{\phi_x\}^T \{\phi_a\} = 0$$

MAC=0

Proportionality $\{\phi_x\} = k\,\{\phi_a\}$

$$MAC(a,x) = \frac{\left|k\{\phi_a\}^T \{\phi_a\}\right|^2}{\left(k\{\phi_a\}^T k\{\phi_a\}\right)\left(\{\phi_a\}^T \{\phi_a\}\right)} = 1$$

General case

0 < MAC < 1

- Modal Assurance Criteria (MAC)

MAC and the Coherence function – Similarities

Coherence function

$$\gamma^2(f) \equiv \frac{|G_{AB}(f)|^2}{G_{AA}(f) \cdot G_{BB}(f)}$$

Coherence function
Expresses the degree of linearity between signal A and B

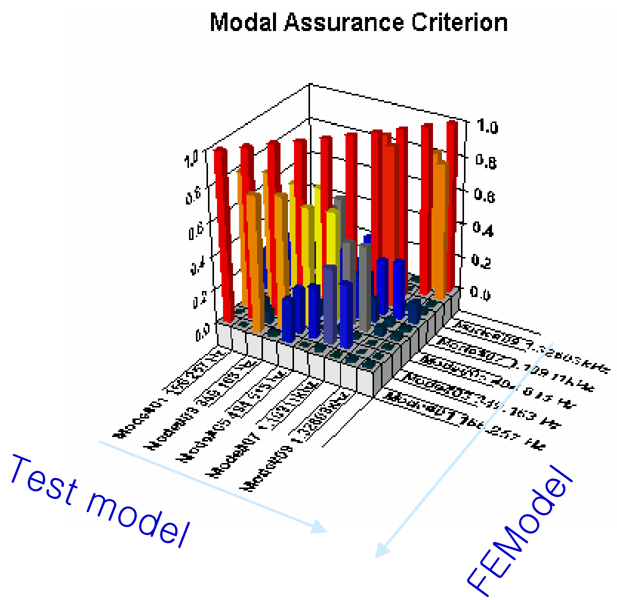
MAC function

$$MAC\ (a,x) = \frac{\left| \{\phi_x\}^T \{\phi_a\} \right|^2}{\left(\{\phi_x\}^T \{\phi_x\}\right)\left(\{\phi_a\}^T \{\phi_a\}\right)}$$

MAC function
Expresses the degree of linearity between modeshape *a* and *x*

MAC matrix

- Tabular or graphic plot of MAC values
- Element $MAC_{a,x}$ denotes MAC(a,x)



4.3 Modal Testing

- Modal Assurance Criteria (MAC)

Interpretation of the MAC function

MAC << 1

- Non stationary
- Non-linearity
- Noise on the vectors (often from measurements)
- Improper curve-fitting

MAC close to1 vectors are well correlated but may be

- Incompletely measured
- Result of harmonic excitation (rotating machinery etc.)
- Result of coherent noise (bias from the estimation)

- Modal Assurance Criteria Auto MAC

MAC matrix properties

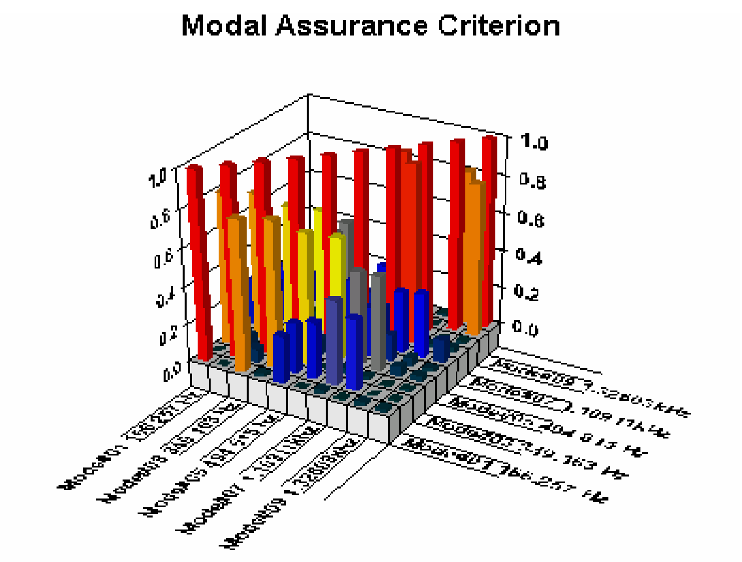
- Diagonal terms equals 1, $MAC(\{\phi_r\},\{\phi_r\}) = 1$
- Off diagonal terms:
  - Modeshapes are orthogonal with respect to mass and stiffness matrix
  - Off diagonal terms > 0 thus indicates a imperfect model

Orthogonality Property

$$\{\phi_r\}^T [M] \{\phi_s\} = 1 \; if \quad r = s$$

$$\{\phi_r\}^T [M] \{\phi_s\} = 0 \; if \quad r \neq s$$

$$[\phi]^T [M] [\phi] = [\delta]$$



4.4 Structural Dynamic Modification

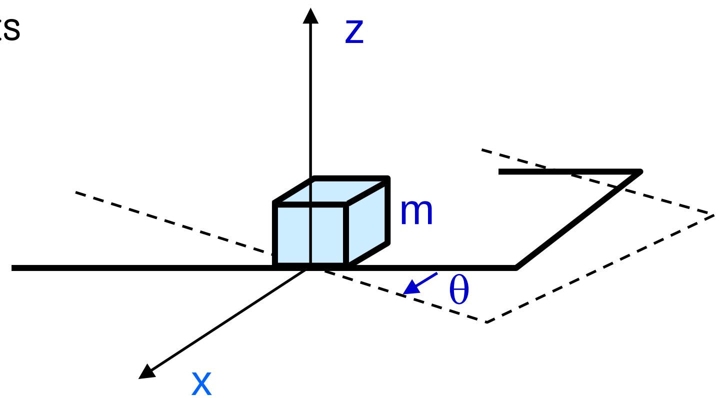
- Purpose of lecture
  - To describe basic physical modifications
  - Modification modelling

- Basic Modification Elements
  - Masses
  - Stiffeners
    - Rigid Links
  - Dampers
  - Tuned Absorbers
  - Ribs, Bars, Rods, Triangles, Quads, Tetras, Prisms, Bricks, etc.

- Hardware Modifications

Assumption: Only point mass, which means no rotational inertia loading

Rotational loading specially relevant near Node Points



Translatoric $$m\frac{d^2z}{dt^2} = f(t)$$

Rotational $$I\frac{d^2\theta}{dt^2} = f(t)\cdot r \ \ (\text{Torque})$$

where $I = \int_m r^2 dm$ is the moment of inertia of mass around x

Rotational DOFs in Model are required in order to handle rotational inertia modifications

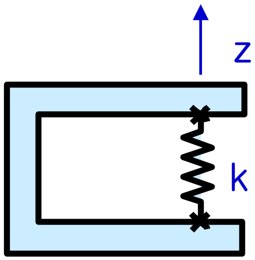
- Hardware Modifications: Mass
  - Point mass *added* or *removed* from *a point* or *a DOF* in Modal Model
  - Notice that only Modal Model and no geometry of structure is required for the SDM calculations

- Hardware Modifications: Stiffness and Damping
  - Between DOFs
    - No geometry is required for these modifications between DOFs
  - Between Points
    - The end coordinates are here used in the SDM program.
      The element properties are transformed from local to global coordinate system.

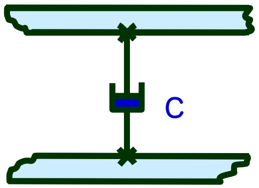
- Hardware Modifications: Stiffness and Damping

Linear springs and viscous dampers can be added or removed between any two points or two DOFs

- Between DOFs



Add 5,000,000 units of stiffness between DOF 14 z and DOF 19 z

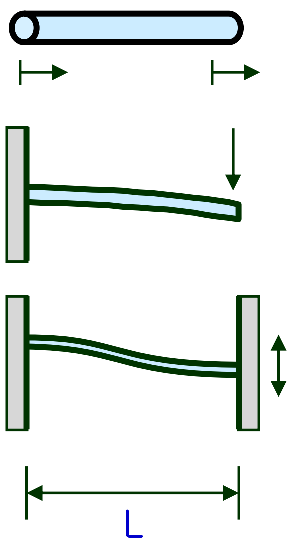


Add 100 units of viscous damping between DOF 5 x and 8 x

Useful for modelling effects in only one direction, which could be axial, shear or bending stiffness of a beam

- Hardware Modifications: Modeling of Stiffeners

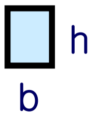
Spring Constants:



$$k = \frac{EA}{L}$$

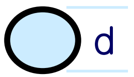
$$m = \rho AI$$

$$k = \frac{3EI}{L^3}$$



$$I = \frac{1}{12} bh^3$$

$$k = \frac{12EI}{L^3}$$



$$I = \frac{1}{64} \pi d^4$$

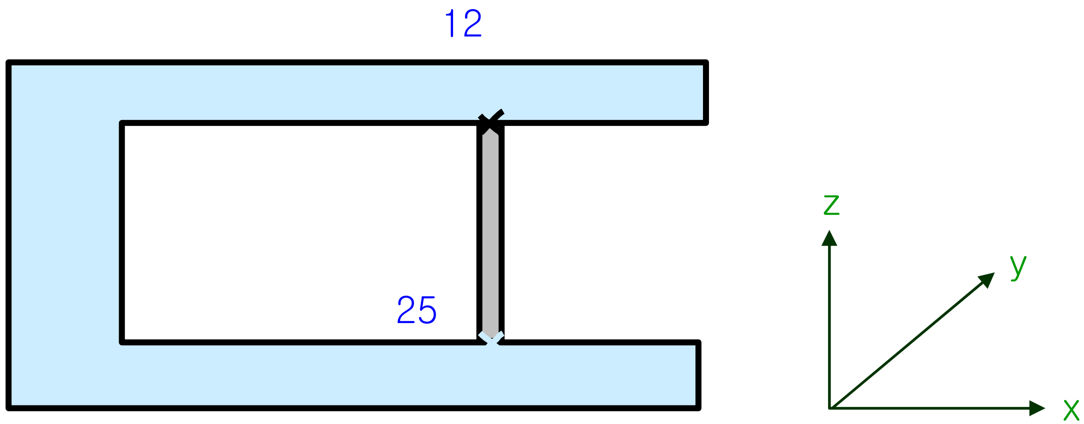
Steel: $E = 2.1 \times 10^{11}$ [N/m²]

$1\ [N/m^2] = 1\ [Pa]$

$\rho = 7.85 \times 10^3$ [kg/m³]

- Hardware Modifications: Rigid links

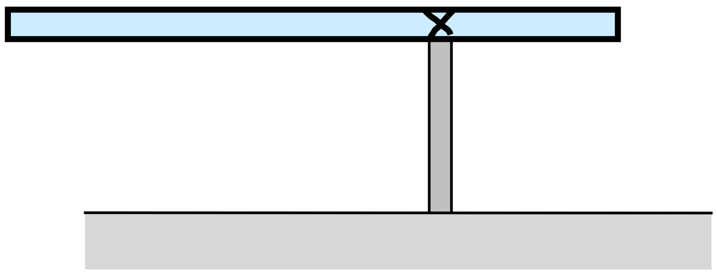
  - Rigid links = Infinite stiffness
  - Rigid links can be connected between any two DOF's on the stucture
  - Rigid links will remove one point and one mode from the model.
    Rigid links can therefore *not be removed* since this information is lost



- Hardware Modifications: Connections to Ground

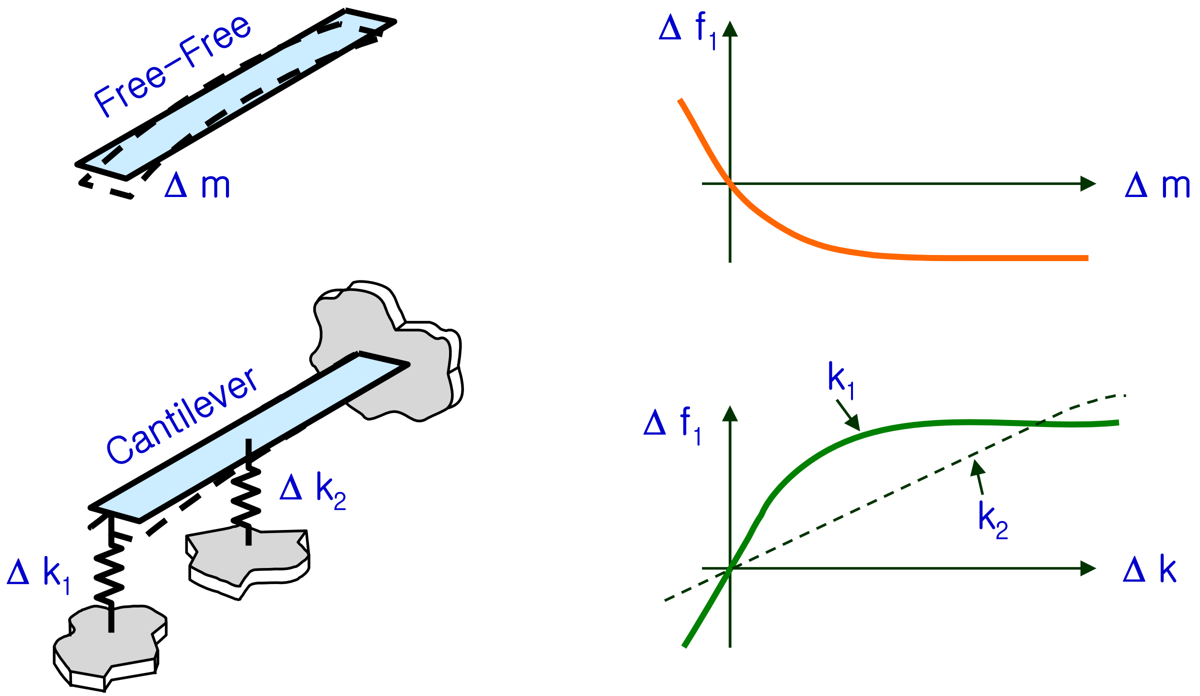
  - Ground is a point without motion
    - Location entered in global coordinates

  - If Point or DOF is connected to Ground with a Rigid Link, it will be a stationary point (hinged)



4.4 Structural Dynamic Modification

- Sensitivity

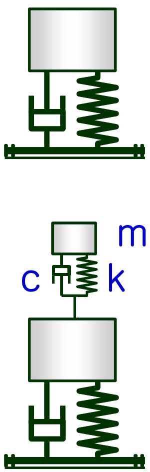


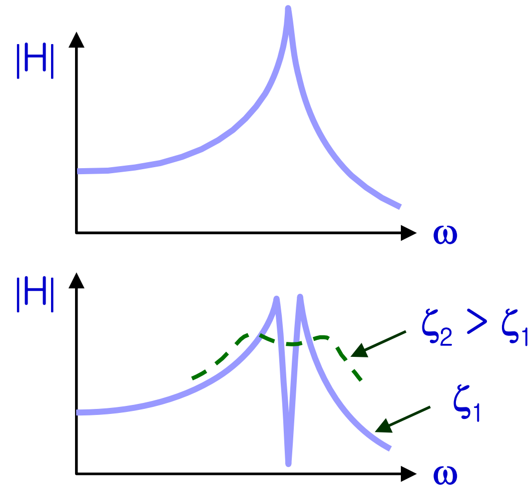
- How much does the natural frequency variy with added mass and stiffners ?

4.4 Structural Dynamic Modification

- Tuned Absorber (TA)

Purpose: To split a mode into two "side modes"





Optional Tuned Absorber Design:

- minimum response at original resonance
- maximum response of side modes are approximately equal

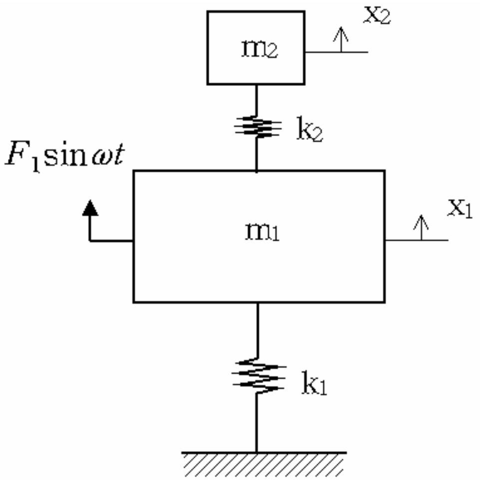
obtained if:

$m_{TA} \sim 5 - 10\%$ of m_A

$\zeta_{TA} \sim \zeta_{resonance}$

$\omega_{dTA} \sim \omega_{dresonance}$

- Dynamic Absorber



$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_o \\ 0 \end{Bmatrix}\sin\omega t$$

$$\text{Let } \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\sin\omega t$$

$$\begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_o \\ 0 \end{Bmatrix}$$

$$[z(\omega)] \equiv \begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \frac{adj\ [z(\omega)]}{det\ [z(\omega)]}\begin{Bmatrix} F_o \\ 0 \end{Bmatrix}$$

$$det\ [z(\omega)] = (k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2) - k_2^2$$

$$adj\ [z(\omega)] = \begin{bmatrix} k_2-m_2\omega^2 & k_2 \\ k_2 & k_1+k_2-m_1\omega^2 \end{bmatrix}$$

$$\therefore\quad X_1 = \frac{(k_2-m_2\omega^2)F_o}{(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2) - k_2^2}$$

- Design method

$\omega = \sqrt{\dfrac{k_2}{m_2}}$ + an optimal damping

$X_2 = \dfrac{F_o}{k_2}$: check point

- Tuned Absorber Implementation

$m_{Ta} \sim 5 - 10\%$ of m_A

$$\alpha_{ii}(\omega) = \sum_{r=1}^{n} \frac{\phi_{ir}\phi_{ir}}{K_r - \omega^2 M_r + j\omega C} = \sum_{r=0}^{N} \frac{{}_rR_{ij}}{\omega_r^2 - \omega^2 + j2\varsigma_r\omega_r\omega}$$

$$\alpha_{ii}(\omega) = \sum_{r=0}^{N} \frac{\psi_{ir}\psi_{ir}}{\omega_r^2 - \omega^2 + j2\varsigma_r\omega_r\omega}$$

The apparent mass (m_A) in a specific point/direction (i) for a specific resonance (r) is:

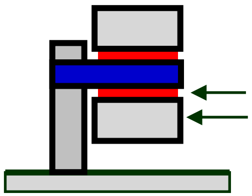
$$m_{Aiir} = \frac{1}{\psi_{ir}^2}$$

Approximate to 1 DOF system

$$k_{TA} = \omega_o^2 \cdot m_{TA}$$

$$c_{TA} = \sigma \cdot 2\, m_{TA} = \zeta_{TA} \cdot \omega_o^2 \cdot m_{TA}$$

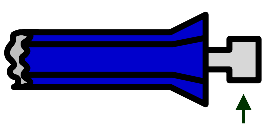
Examples:



Rubber spring/damper

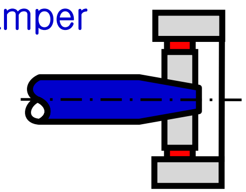
Mass

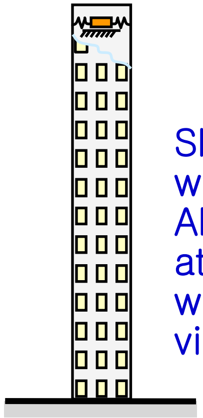
Tennis Racket



Rubber mass/ spring/damper

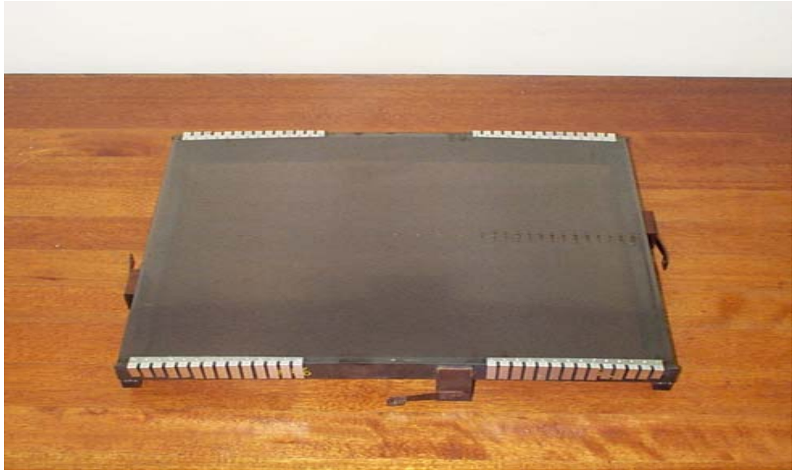
Torsional Damper

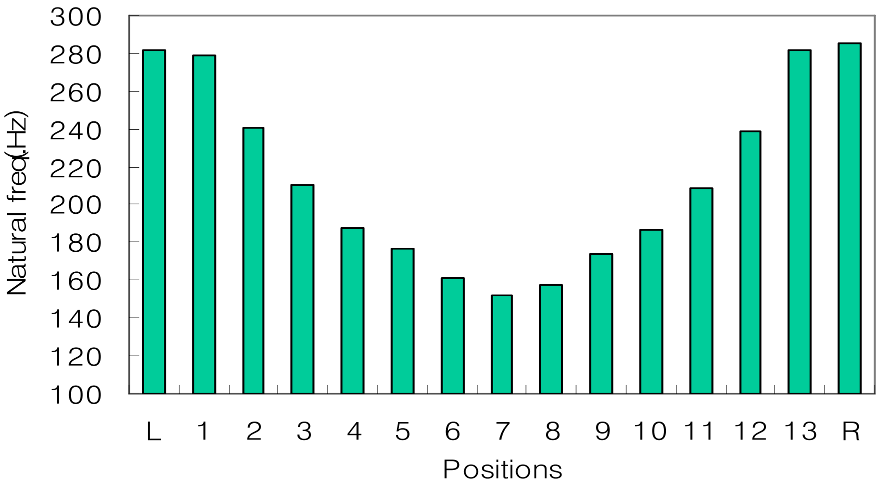




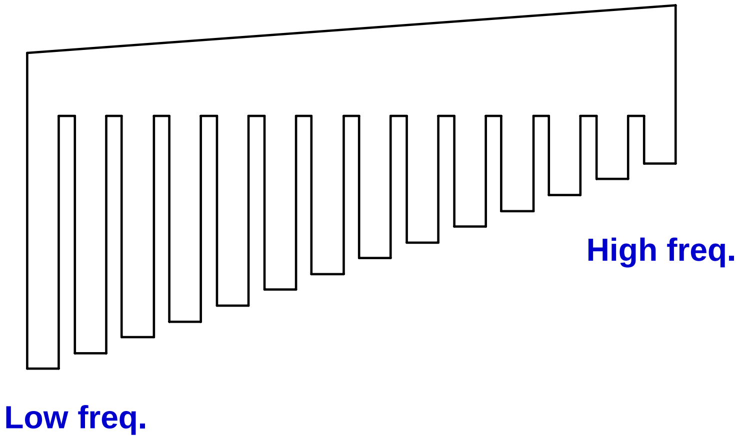
Skyscraper with Tuned Absorber for attenuation of wind excited vibrations

- MDA of Shadow-mask





Multiple Dynamic absorber



- MDA of Shadow-mask

Frequency Tunning

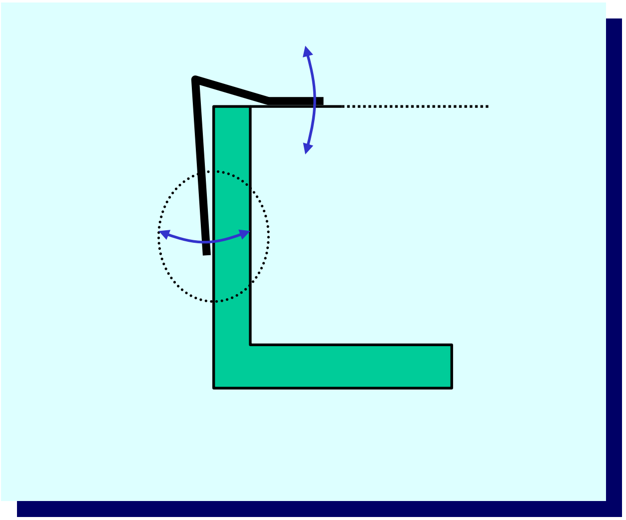
Simple clamped – free beam
(1st mode)

$$\omega_1 = 1.875^2 \sqrt{\frac{EI}{mL^4}} \quad (rad / s)$$

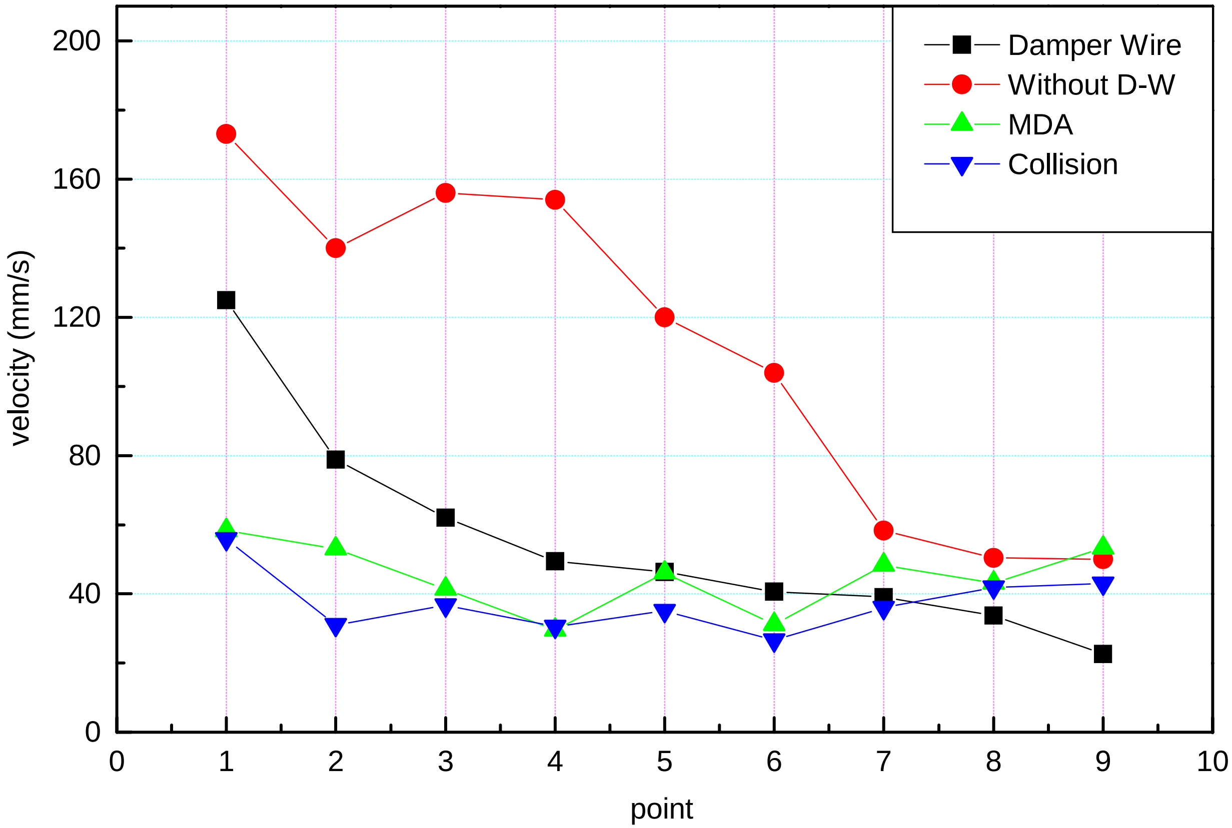
$$, m = mass \ per \ unit \ length$$

$$\therefore \omega_1 \propto \frac{1}{L^2}$$

Damping Mechanism

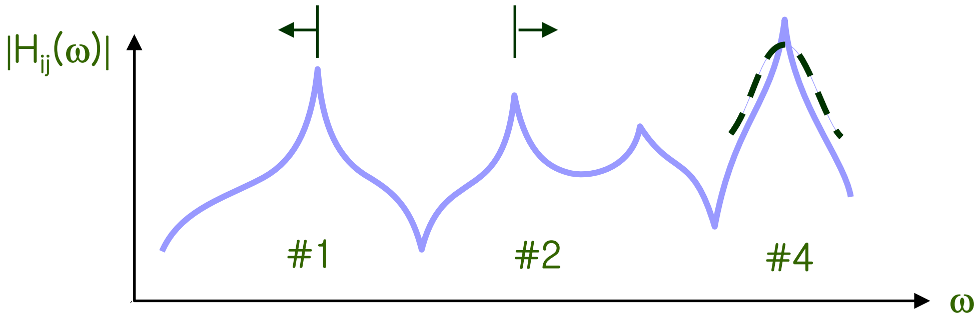


- MDA of Shadow-mask



- Resonance Specification

Situation: Want to change a resonance by a hardware modification



Specify: Resonance
Type of modification and DOF

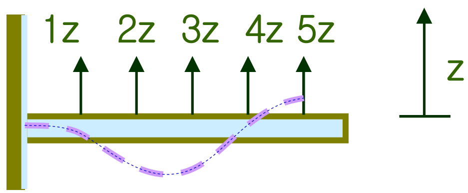
Solution: SDM computes the amount of hardware modification

The hardware modification commands can then be applied
in order to find the new modal parameters

- General Limitation in SDM

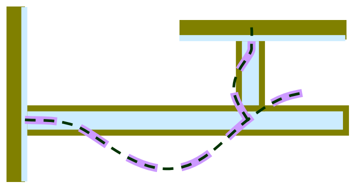
We only take into account the DOFs in the modal model

Example: Cantilever beam only translatory DOFs in z−direction



Mode #2

SDM will not change
Mode #2 in model



Mode #2 would be changed due to constraint of Rotation at point 4

4.5 Forced Response Simulation

- Purpose of lecture

  - To describe the possibilities/limitations in the use of the Response Model
  - To simulate Operational Deflection Shapes

- Response Model

  - Simulate the response of the system due to specified force excitations by use of the linear Modal Model

Modal Model

$$\begin{Bmatrix} X_1(\omega) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ X_n(\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & \cdot & \cdot & H_{1n}(\omega) \\ H_{21}(\omega) & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ H_{n1}(\omega) & \cdot & \cdot & \cdot & H_{nn}(\omega) \end{bmatrix} \begin{Bmatrix} F_1(\omega) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ F_n(\omega) \end{Bmatrix}$$

where each element in [H (ω)] is

$$H_{ij}(\omega) = \sum_{r=1}^{m} \frac{R_{ijr}}{j\omega - (j\omega_{dr} - \sigma_r)} + \frac{R_{ijr}^{*}}{j\omega - (-j\omega_{dr} - \sigma_r)}$$

i.e. given by the Modal Parameters:

$R_{ijr} = a_r \phi_{ir} \phi_{ir}$ residue for mode # r

ω_{dr} = Damped natural frequency for mode # r

σ_r = Decay rate for mode # r

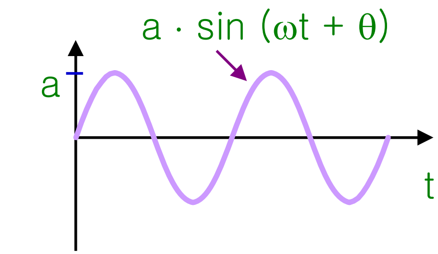
m = Number of modes in model

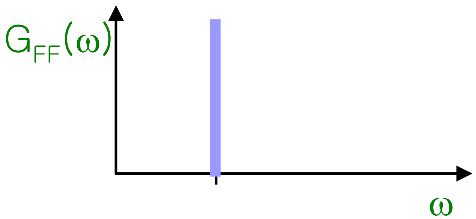
n = Number of DOF's (x, y, z, q_x, q_y, q_z)

4.5 Forced Response Simulation

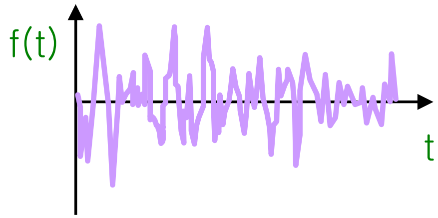
- Types of Excitation Forces

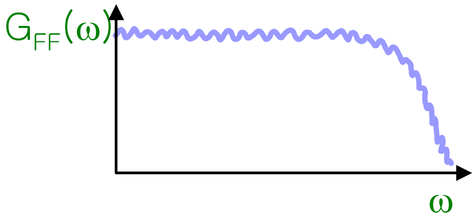
  - Sinusoidal



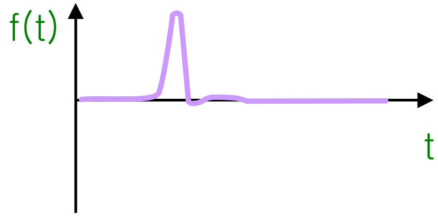


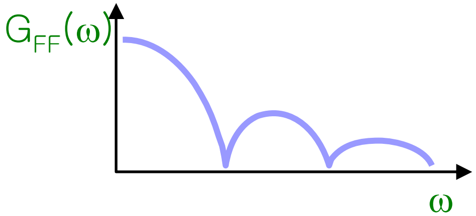
  - Broad banded



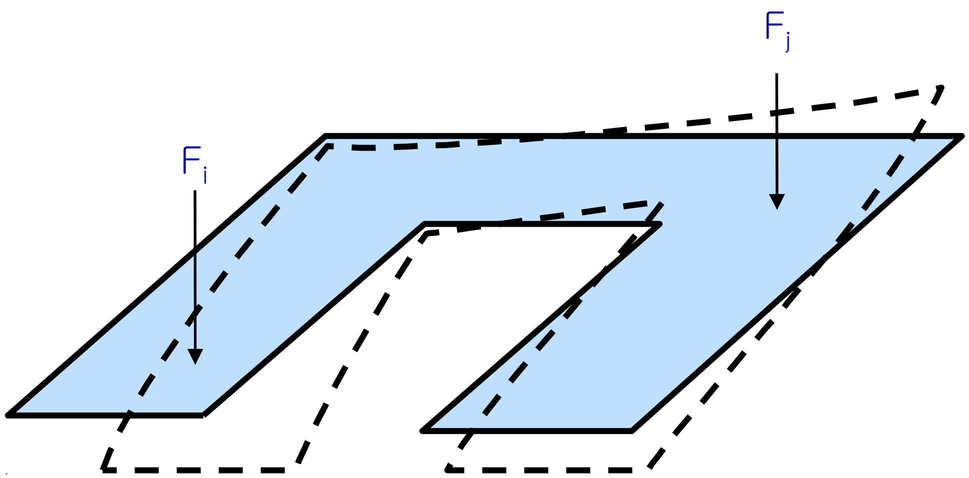


  - Impulse





- Sinusoidal Excitation – Deflection Shape



- The response to a sinusoidal excitation is a forced deflection described as a linear combination of all Mode Shapes

- Sinusoidal

$$Sinusoidal\ \ excitation\ \ A_i\, e^{(j\omega_i t+\theta_i)}\ \ at\ one\ or\ more\ DOFs$$

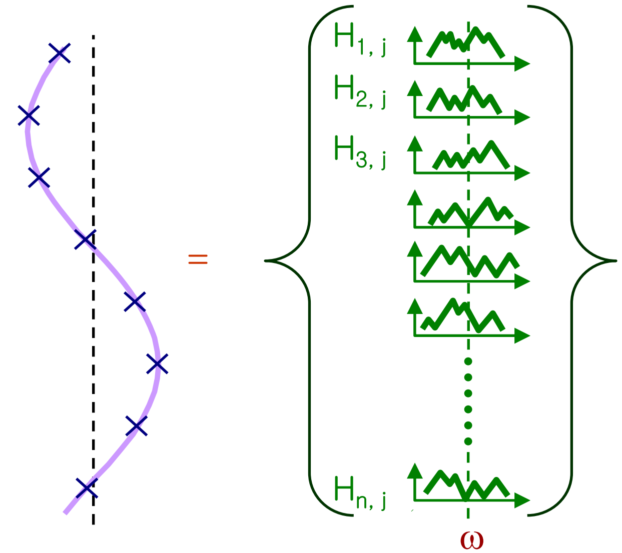
  - The Deformation Shape of the structure is computed and can be animated
    - Same frequencies ω_i at all DOFs
    - A_i and θ_i no restrictions

$$\{F(\omega)\} = \begin{Bmatrix} . \\ . \\ A_i e^{j\theta i} \\ . \\ . \\ A_i e^{j\theta i} \\ . \\ . \end{Bmatrix} e^{j\omega_i t} \quad \xrightarrow[\text{Model}]{\text{Modal}} \quad \begin{Bmatrix} X_1 \\ . \\ X_i \\ . \\ . \\ . \\ . \\ X_n \end{Bmatrix} e^{j\omega_i t}$$

The Deformation Shape

- Sinusoidal Excitation at Single DOF j

$$\begin{Bmatrix} X_1 \\ \cdot \\ X_i \\ \cdot \\ \cdot \\ X_n \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & \cdot & H_{1j}(\omega) & \cdot & H_{1n}(\omega) \\ \cdot & & & & \cdot \\ H_{i1}(\omega) & & H_{ij}(\omega) & & \cdot \\ \cdot & & & & \cdot \\ H_{n1}(\omega) & \cdot & H_{nj}(\omega) & \cdot & H_{nn}(\omega) \end{bmatrix} \begin{Bmatrix} 0 \\ \cdot \\ A_j \\ \cdot \\ \cdot \\ 0 \end{Bmatrix} e^{j\omega t}$$



j^{th} column

- For Multiple inputs, {X} is sequentially calculated:

$$\{X\} = \{H\}_j A_j e^{j\theta_j} + \{H\}_k A_k e^{j\theta_k} +$$

- Response Model

  - Single Degree of Freedom – SDOF

$$X = H \cdot F$$

  - Multi Degree of Freedom – MDOF

$$\{X\} = [H] \cdot \{F\}$$

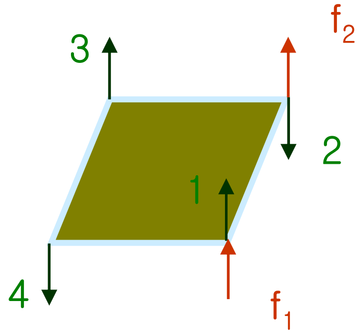
$$\begin{aligned} X_1 &= H_{11} \cdot F_1 + \\ &\quad H_{12} \cdot F_2 + \\ &\quad H_{13} \cdot F_3 + \\ &\quad \ldots \end{aligned}$$

  - Operational Deflection Shape depends on Force Distribution
    i.e. the Generalized Force or Modal Force

- Generalized Force

  - The Generalized Force Γ_1 describes how efficiently a particular force distribution excites a particular mode
  - Example: # 1 mode on plate

$$0 = \{1, -1, 1, -1\} \cdot \begin{Bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$



The mode is not excited !!

Generalized Force = Mode shape × Force distribution

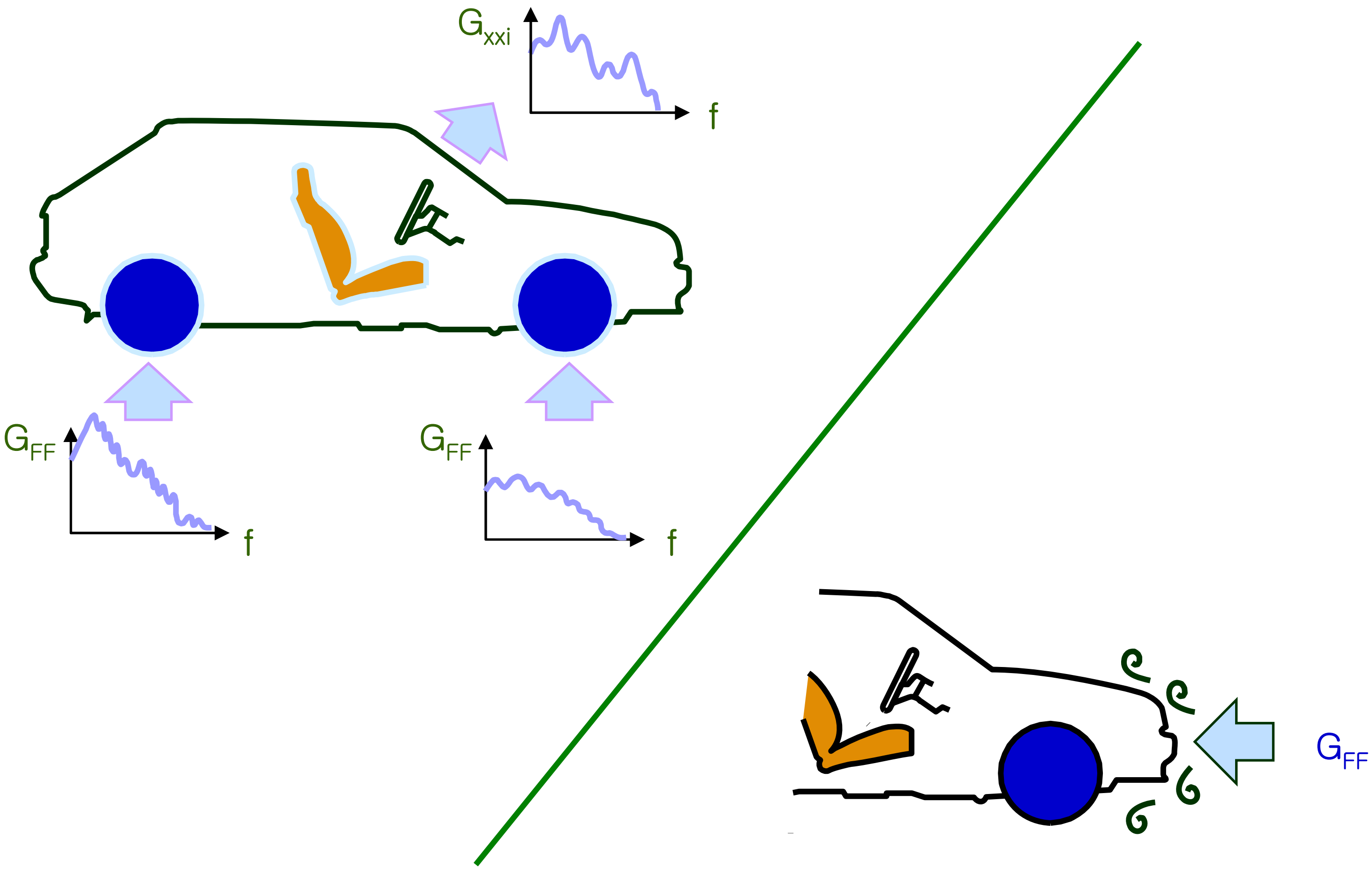
  - Γ_1 = Generalized force for mode # r

$$\Gamma_r = \sum_i \phi_{ir} \cdot f_i$$

$$\Gamma = [\phi]^T \{f\}$$

$$\Gamma = \begin{bmatrix} \phi_{11} & \phi_{21} \\ \phi_{12} & \phi_{22} \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

- Broadband Excitation



- Broadband Excitation at More DOF's

Uncorrelated inputs

$$G_{xxi}(\omega) = \sum_{j} \left| H_{ij}(\omega) \right|^2 G_{FFj}(\omega)$$

Input Spectrum (Spectra)

- Measured and transferred from analyzer
- Set by block commands

4.6 Operational Deflection Shapes

- Purpose of lecture

  - To show how to measure the vibration pattern of a machine in operating condition

  - To show how Computer Aided Operational Deflection Shapes measurements are performed

- Identifying Noise and Vibration Problems

  Two approaches:

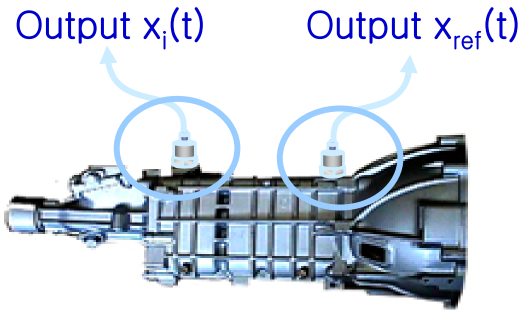
  - Signal Analysis = Operational Deflection Shapes
  - System Analysis = Modal Analysis

  Note:
  Dynamic Modal Model only from Modal Analysis

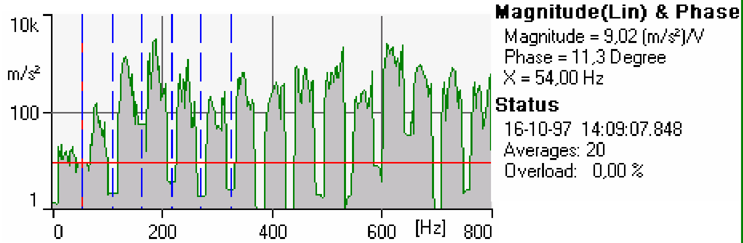
- Operational Deflection Shapes, ODS

  - Two methods

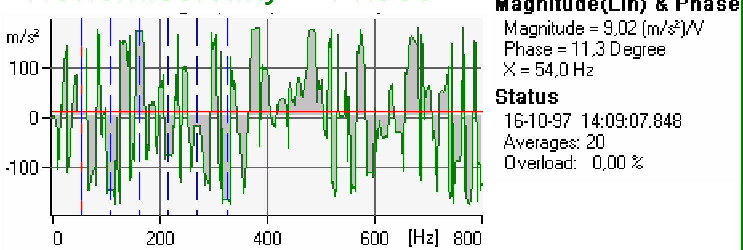
1. Using Transmissibility ($T = x_i/x_{ref}$) giving a relative measurement



Transmissibility – FRF

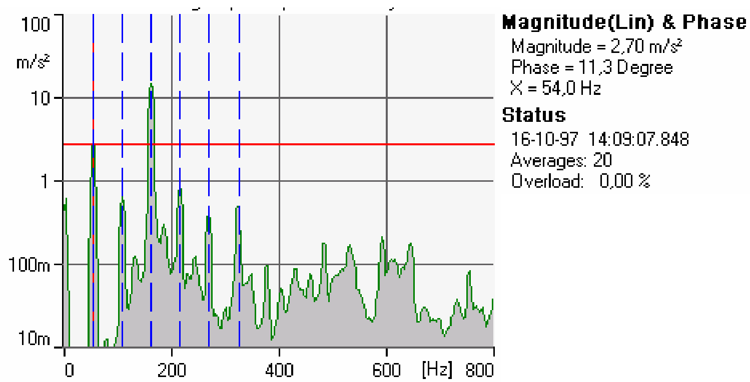


Transmissibility – Phase



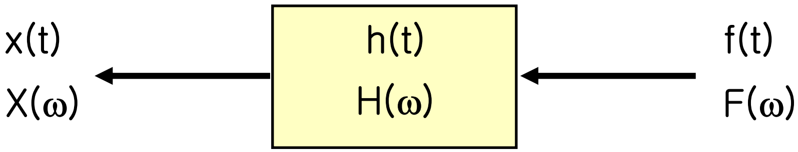
2. Using Phase Assigned Autospectrum (G_{xx}) giving an absolute measurement

Phase Assigned Autospectrum



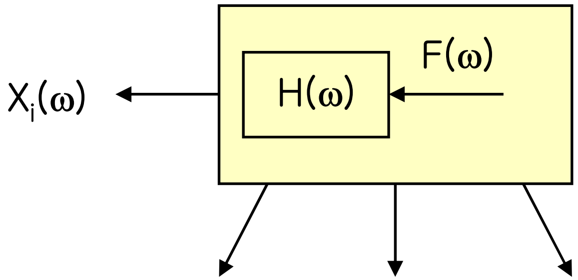
- Operational Deflection Shapes

Dynamic Model (not parametric)



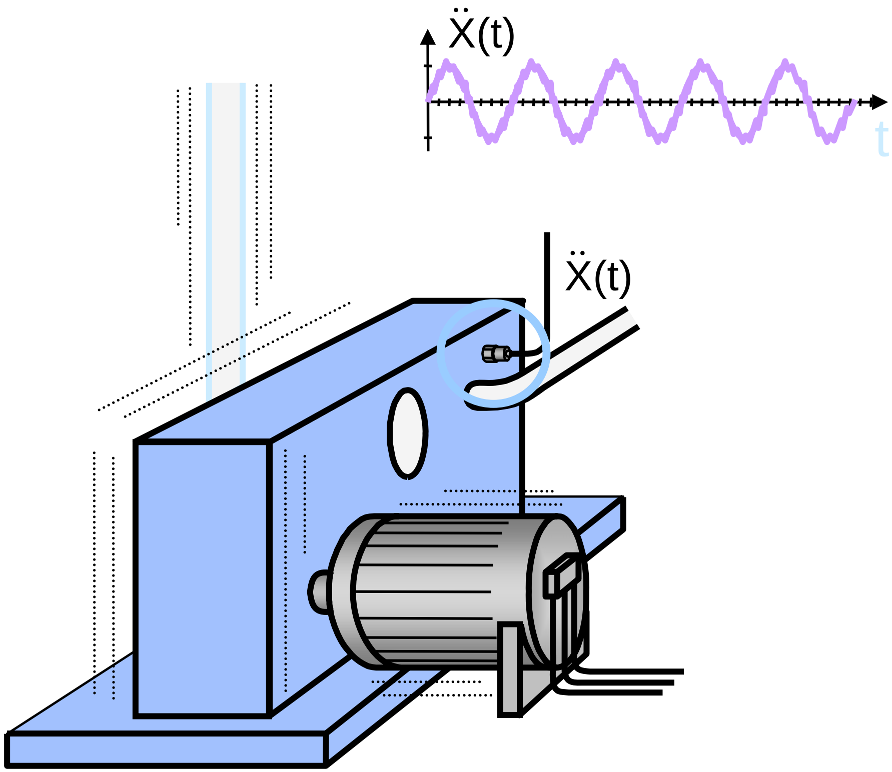
$$X(\omega) = H(\omega) \cdot F(\omega)$$

System and input forces not observable



$\{X_i(\omega)\}$ Observable responses

- Operational Deflection Shapes
    - Operational Condition
        - stationary
        - almost deterministic



- Operational Deflection Shapes

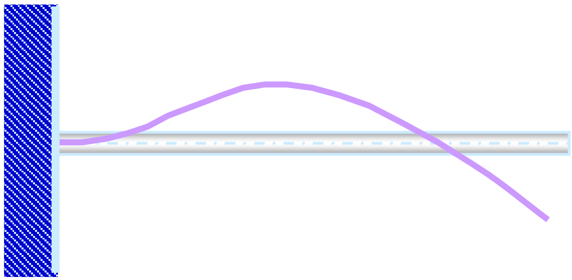
  - Step-by-step Experimental Procedure

1. Setup
   - Decide test point and directions
   - Setup analyzer/transducers
   - Calibrate measuring chain
   - Make trial measurements
   - Select references point

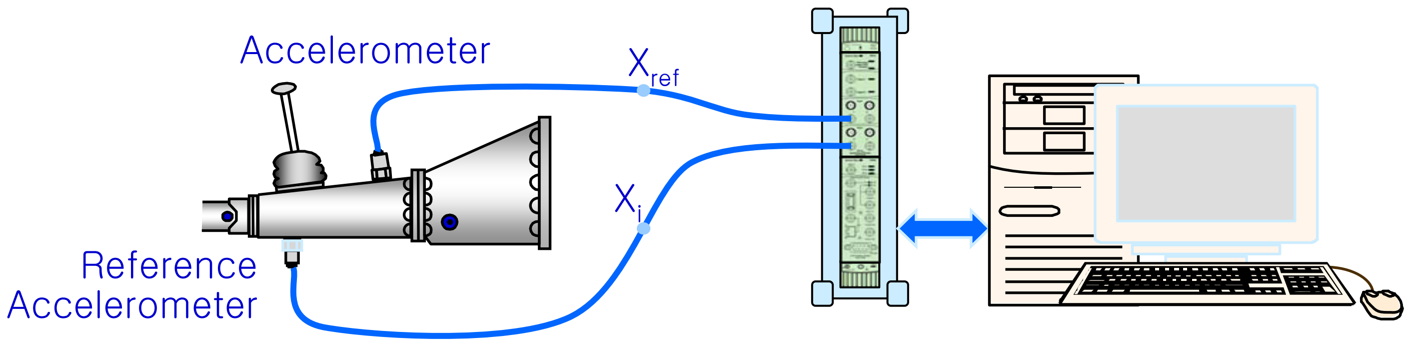
2. Measurements
   - Measure vibration level at reference point
   - Measure transmissibility functions

3. Estimation of Deflection Shapes
   - Determine amplitude ratio and phase difference at running speed or harmonics for all transmissibility functions

4. Draw Deflection Shapes



- Operational Deflection Shapes, ODS
  - Goal: Determine the forced *dynamic deflection* at the *operating speed*

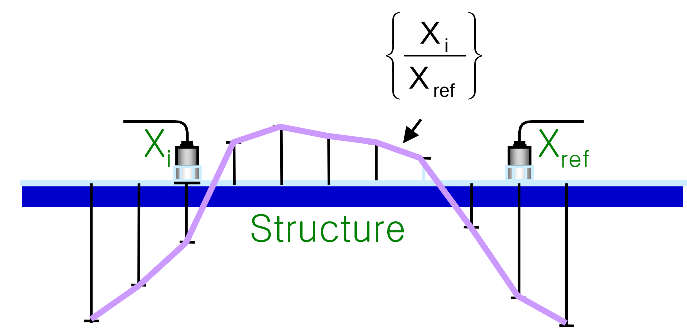


Relative

$$\left\{\frac{X_i}{X_{ref}}\right\}_{f_0} = \begin{Bmatrix} 1.2, & 0^\circ \\ 1.1, & 0^\circ \\ 0.6, & 0^\circ \\ 0.5, & 180^\circ \\ 0.6, & 180^\circ \\ 0.4, & 180^\circ \\ 0.3, & 0^\circ \\ 1.0, & 0^\circ \\ 1.3, & 0^\circ \end{Bmatrix}$$

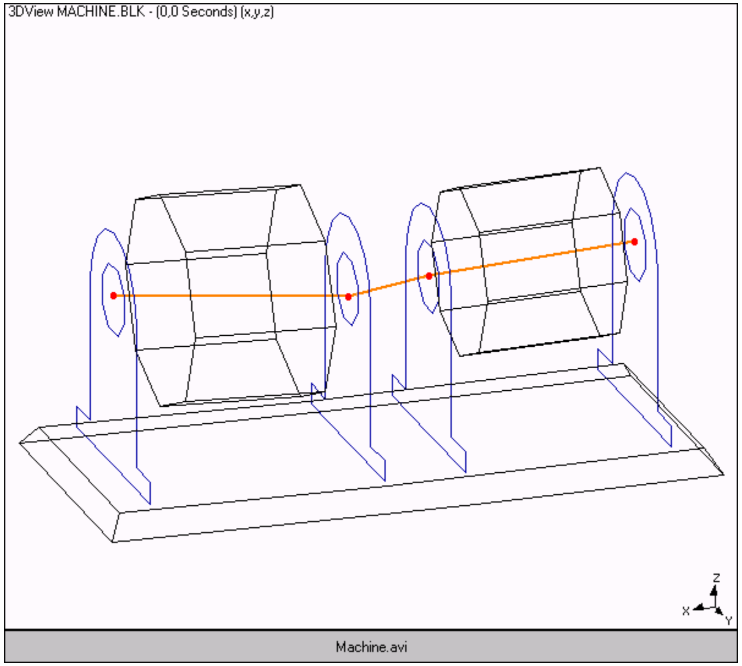
Absolute

$$\{X_i\}_{f_0} = \left\{\frac{X_i}{X_{ref}}\right\} X_{ref}$$

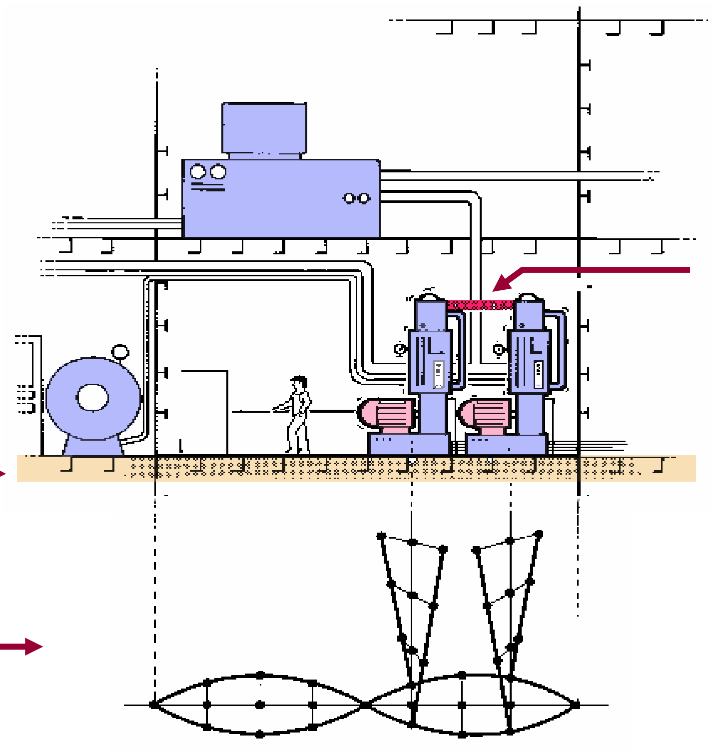


• Computer Aided Operational Deflection Shapes

- Use Modal or dedicated software
- Use Peak Curve Fitting
- Animation of Deflection Shapes
- Relative or absolute motions



- Typical Application of ODS
  - Super Tanker

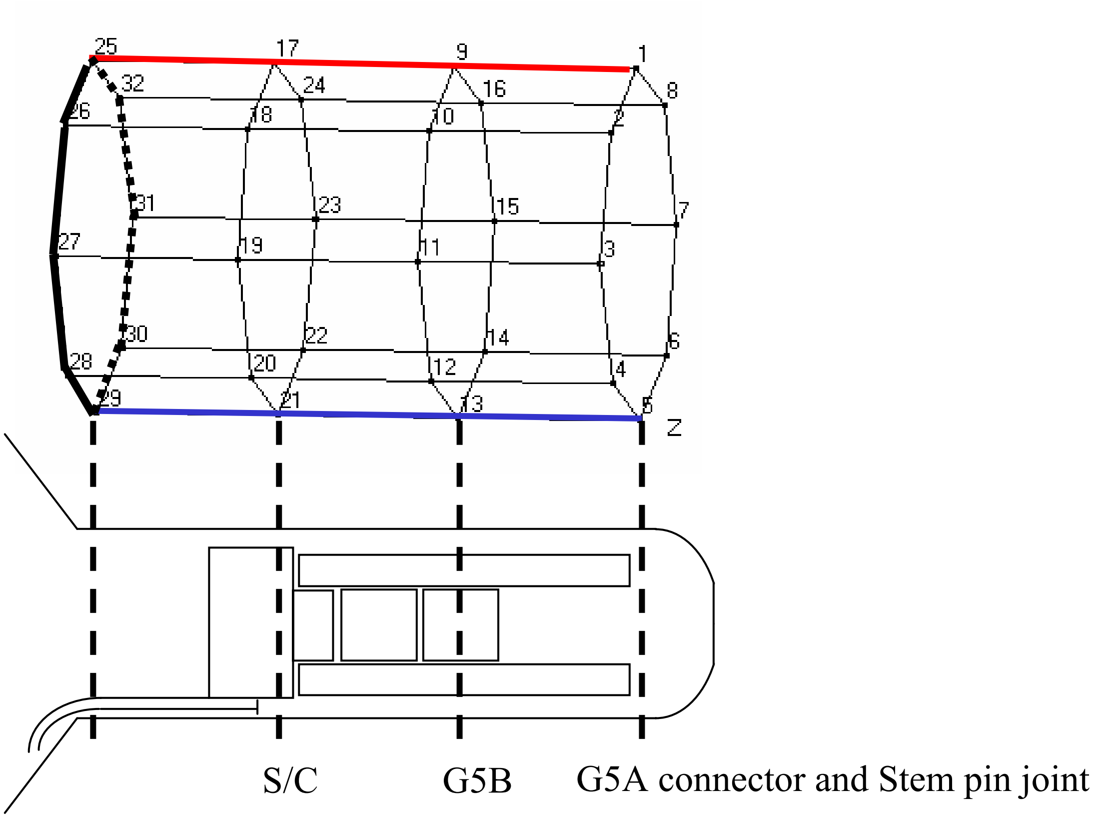


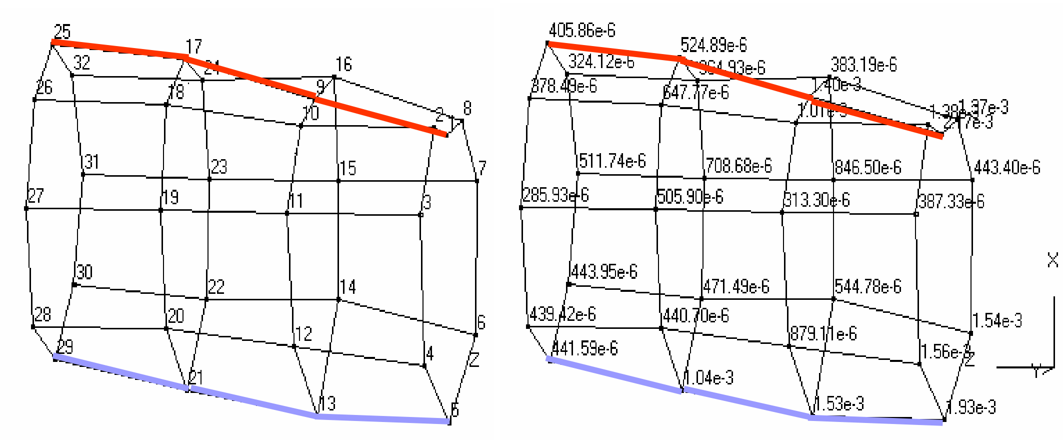
1. First modification proposal: 10 tons of transverse girders for deck stiffening
2. Operational Deflection Shape at 2nd harmonic frequency
3. Optimal modification by rod connection of engine top

- Neck-glass of CRT

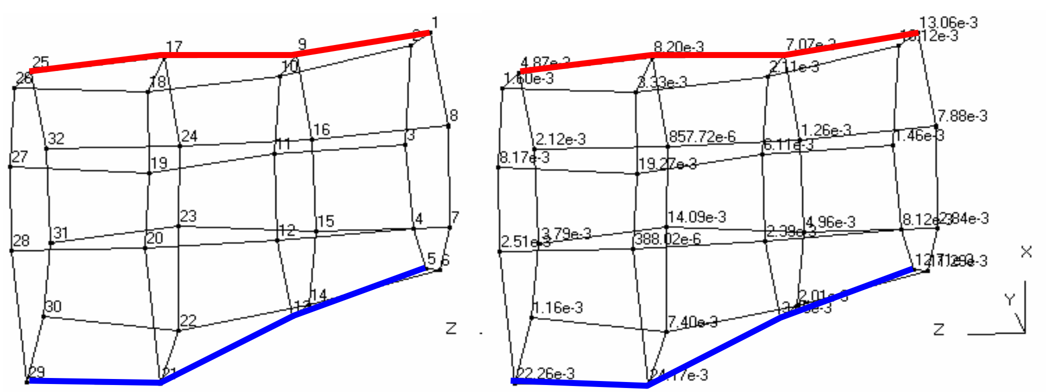
AC Excitation : 3.21 kHz

⇒ 3.21, 4.49, 6.59, 7.65 kHz (composite noise)

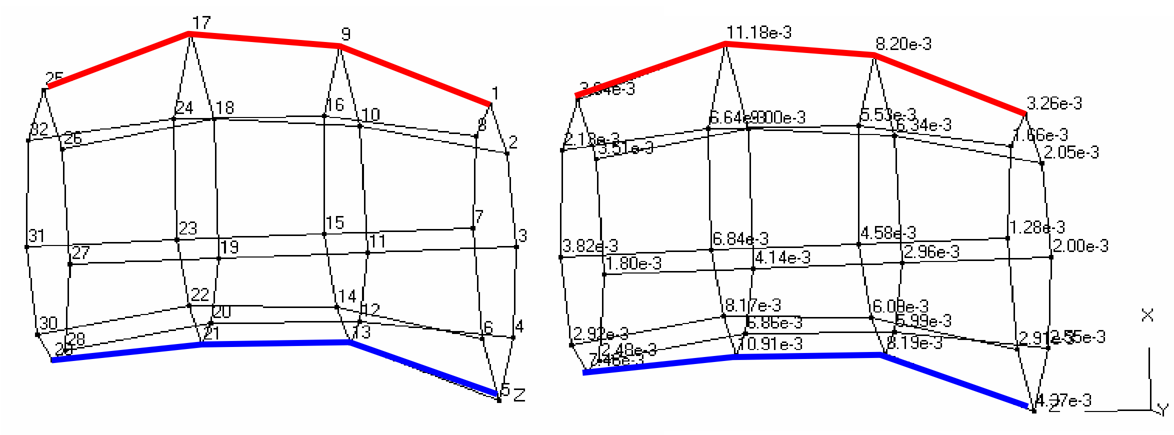




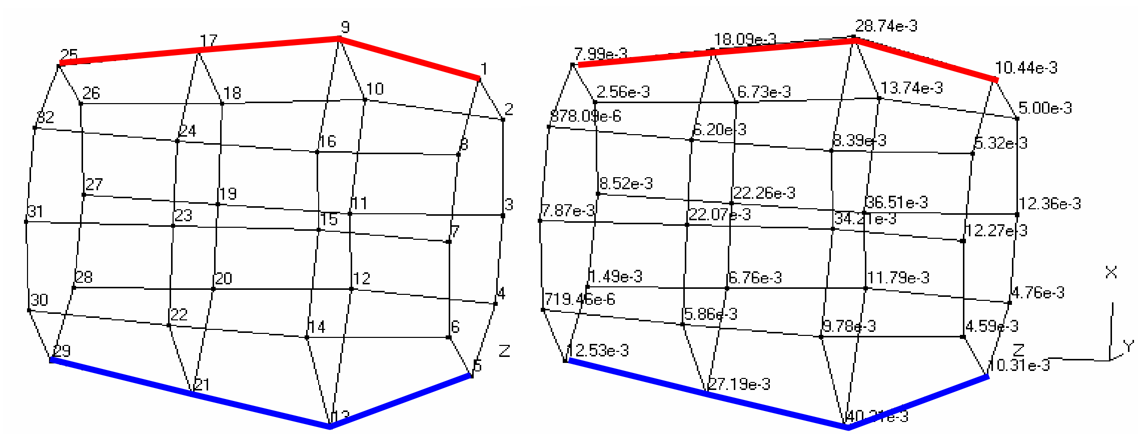
< ODS at 3.21 kHz>



< ODS at 4.49 kHz>



< ODS at 6.59 kHz>



< ODS at 7.65 kHz>

- Operational Deflection Shapes

  - Features:
    - direct measurement of the deflection shape at relevant frequencies under operating conditions
  - Advantages:
    - actual operating forces
    - realistic boundary conditions
    - fast and accurate
    - no assumption of a linear model
  - Benefits:
    - gives the deformation at important frequencies
    - visualisation of deformation shape
      $\Rightarrow$ better understanding $\Rightarrow$ solution

Note: No model obtained $\Rightarrow$ no prediction of responses at other conditions