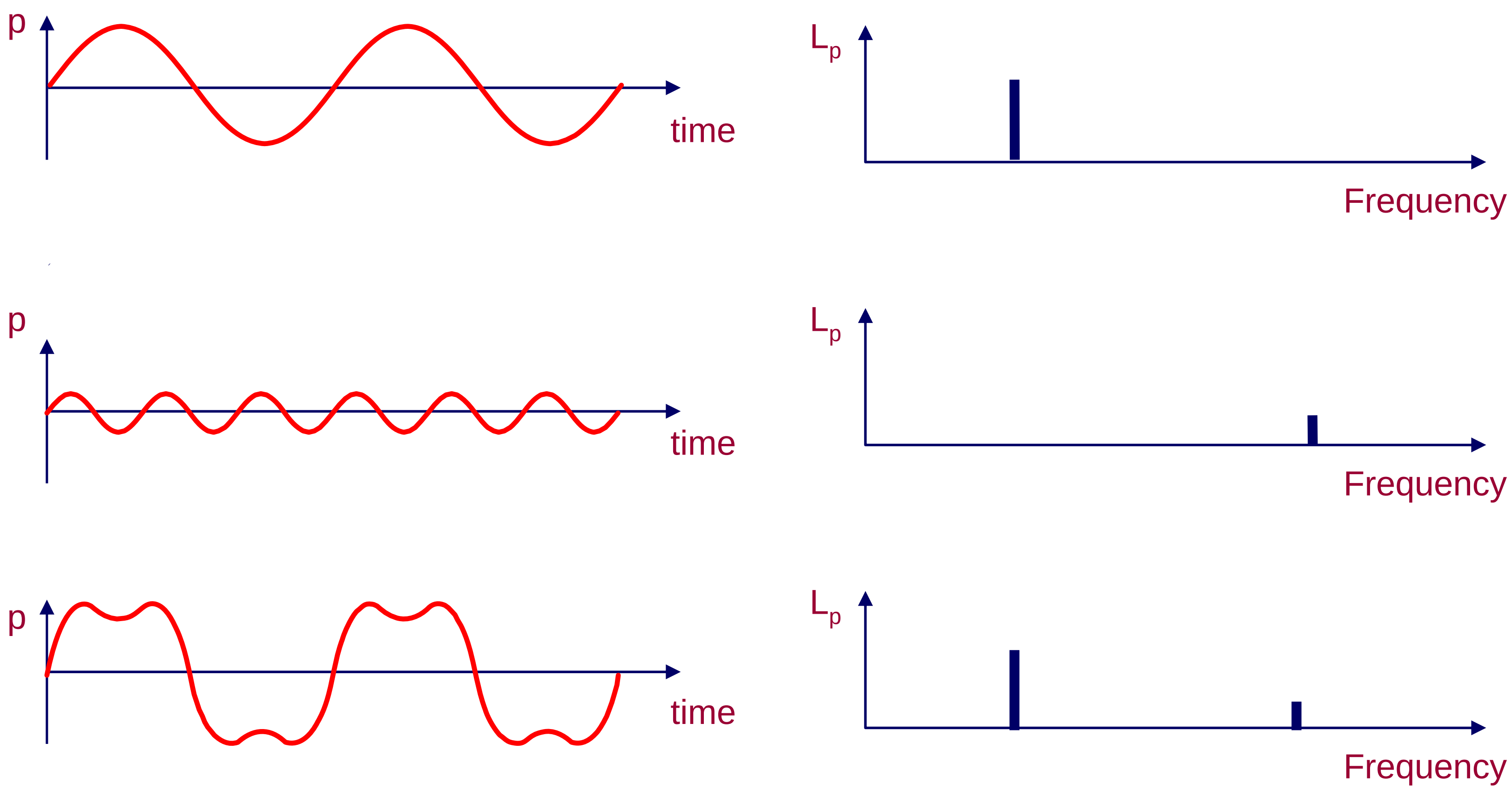
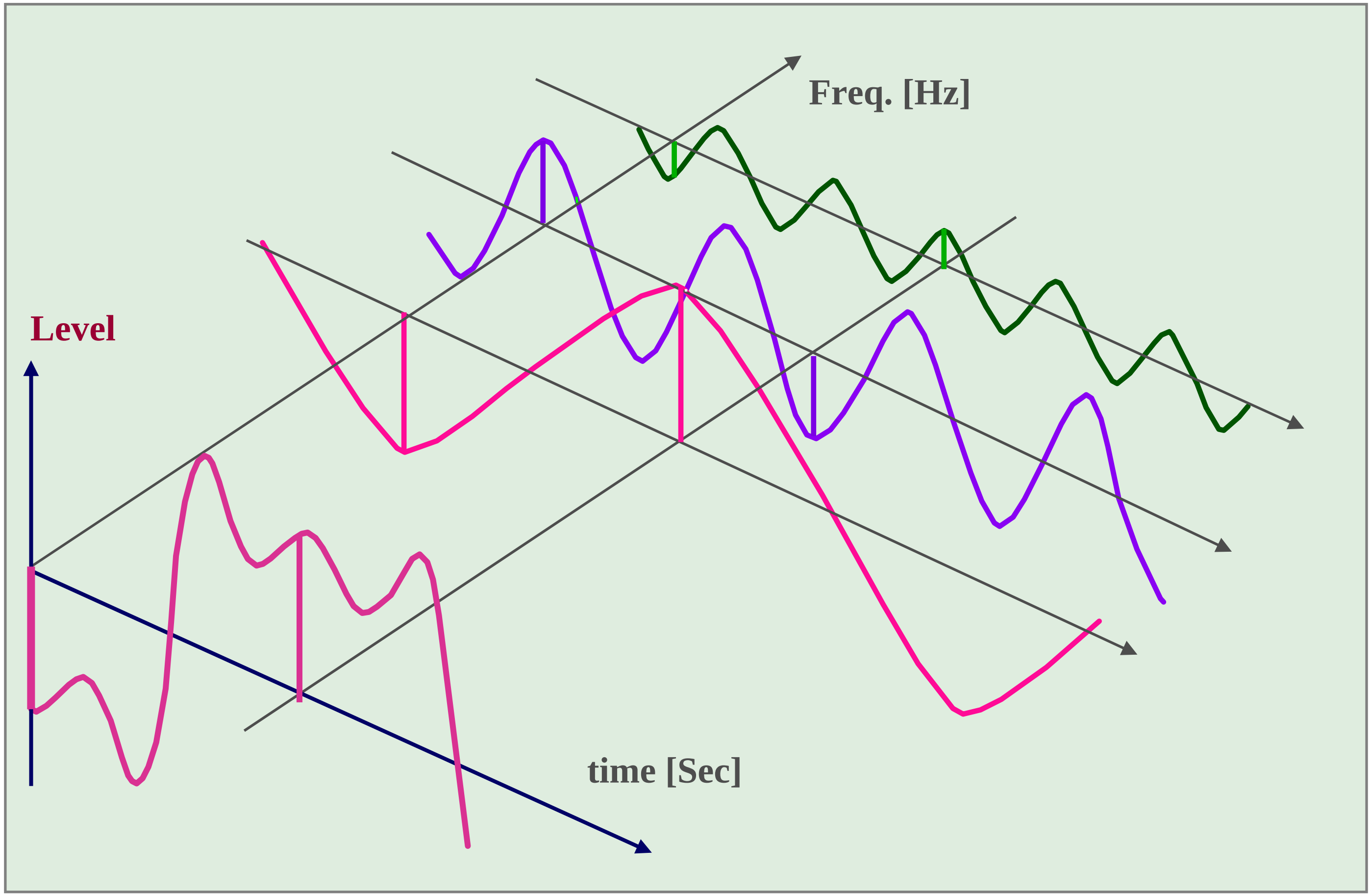
Contents

Slide no.

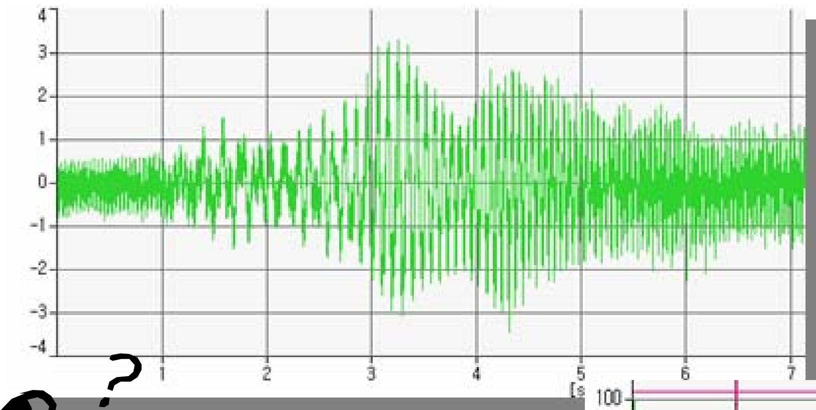
- Waveforms and Frequencies



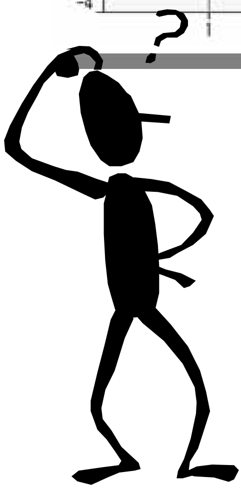
- Time signal & Frequency

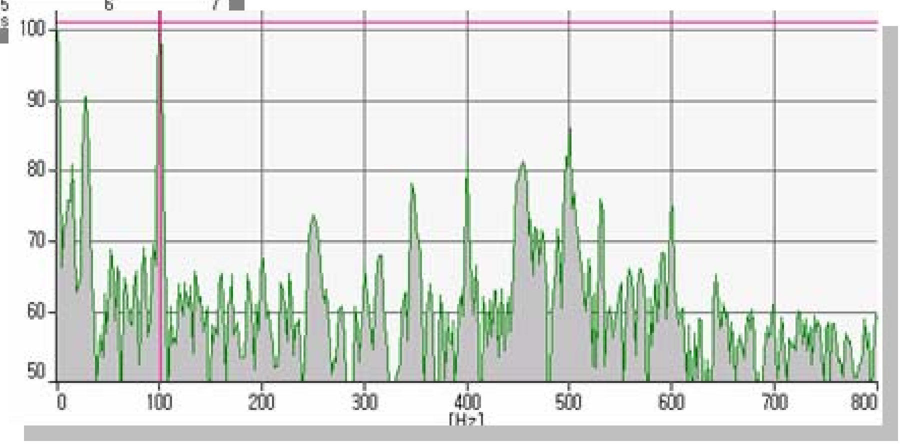


- Signal in time domain & Frequency domain



Time domain

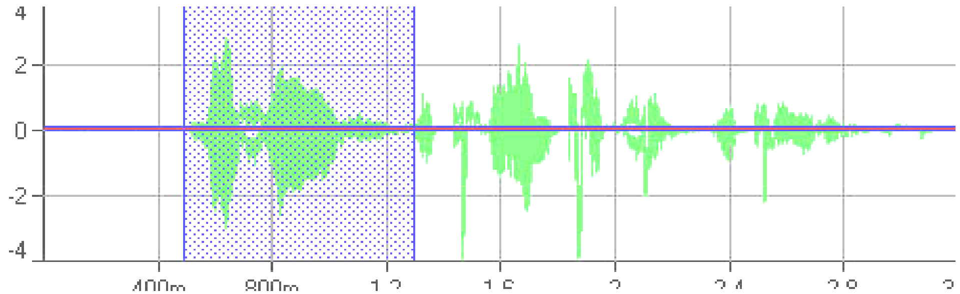




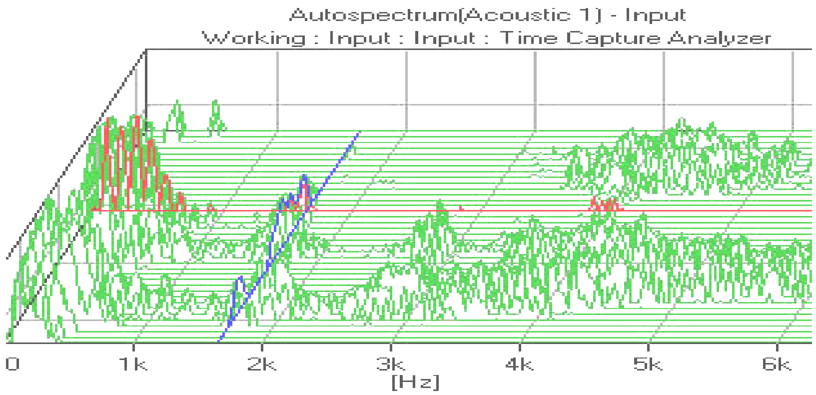
Frequency domain

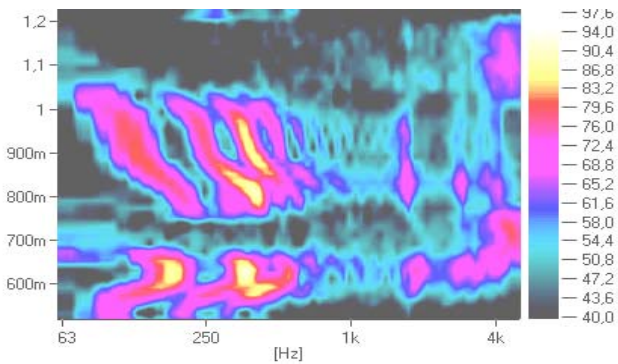
- Time domain & Frequency Domain

  - Time domain
    - Overall Level
    - Analysis of signal wave
    - Probability analysis
    - Oscilloscope

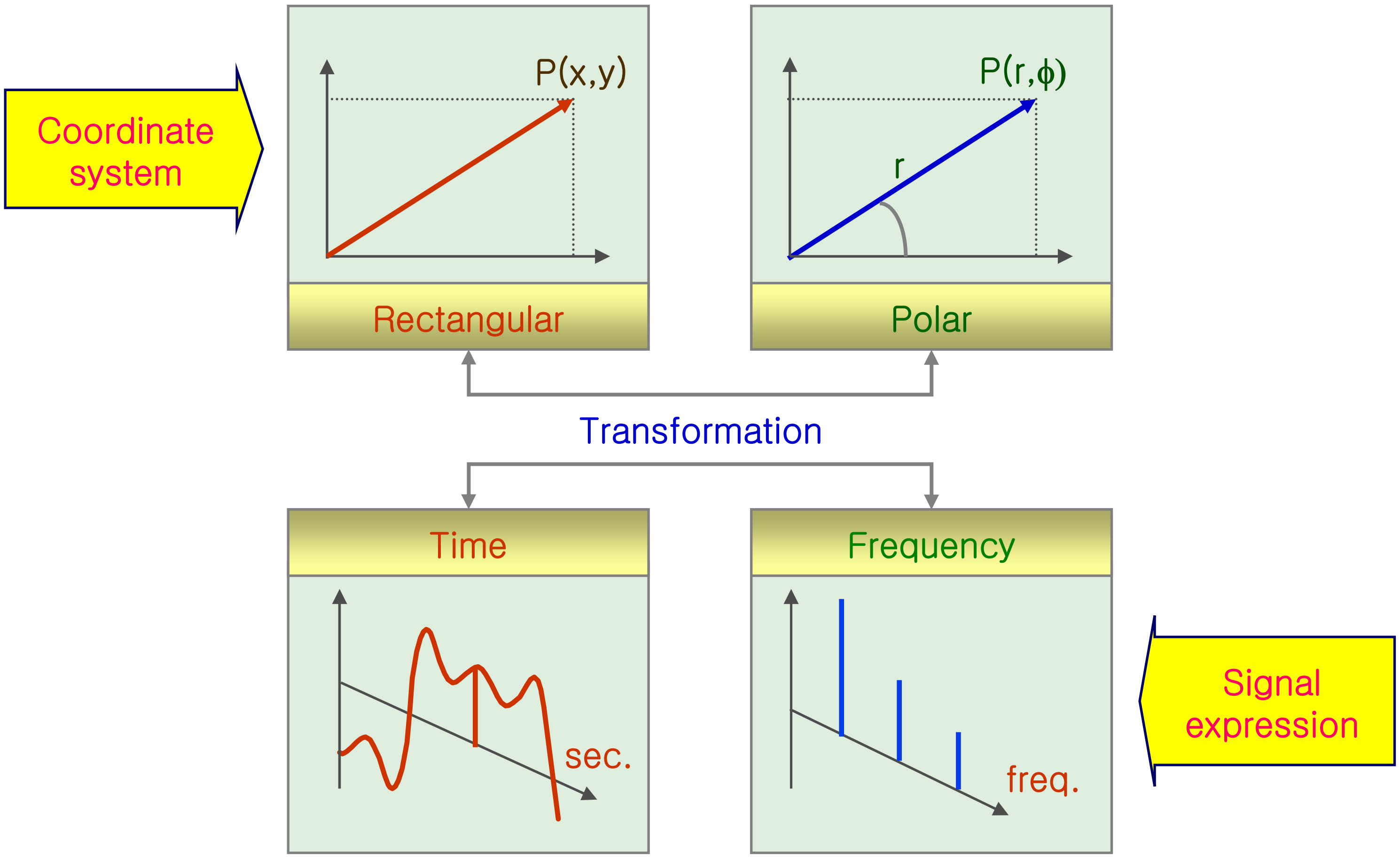


  - Frequency domain
    - Every Frequency contributions for each frequency component

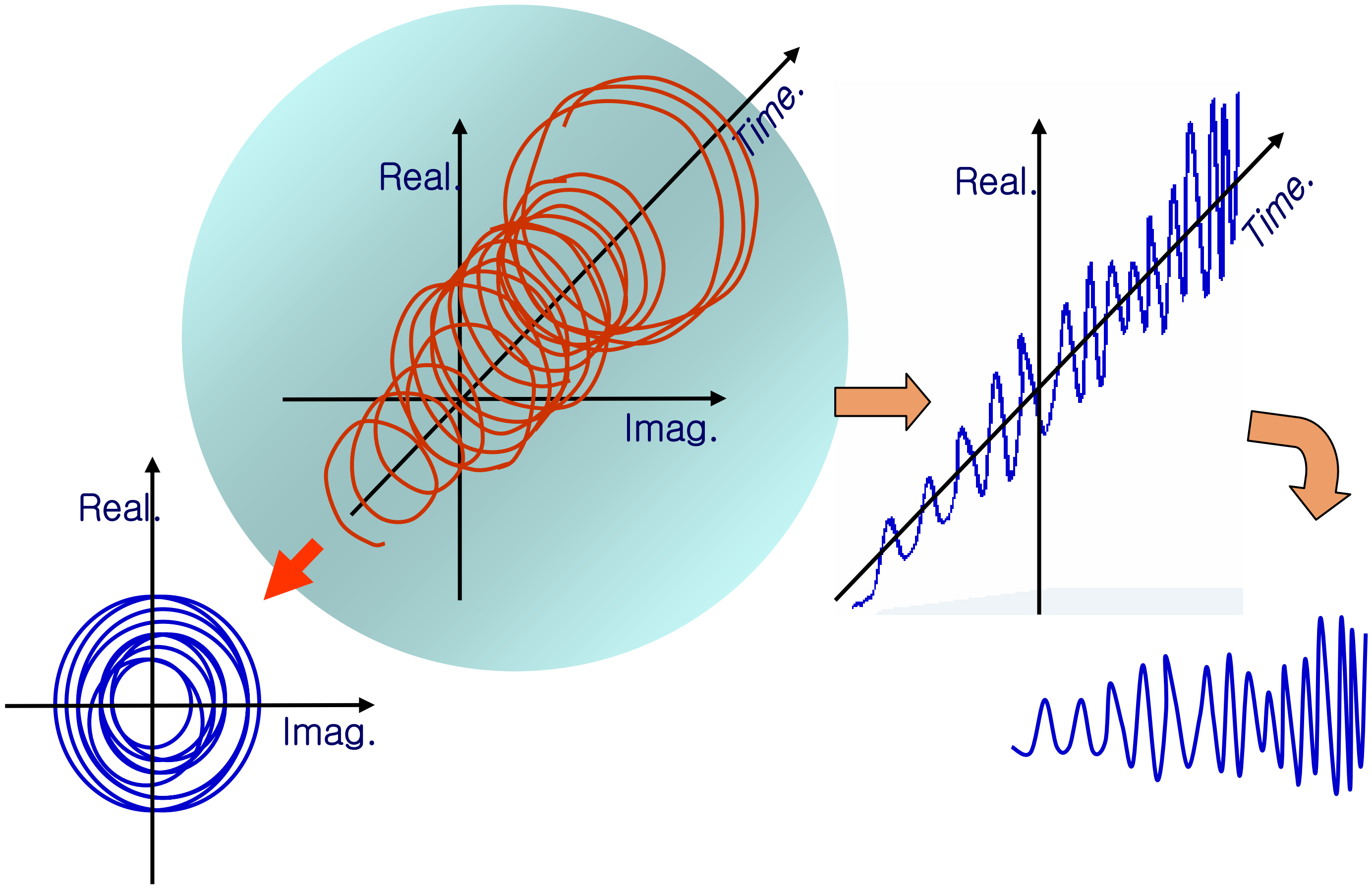




- Coordinate Example



- Coordinate time & frequency



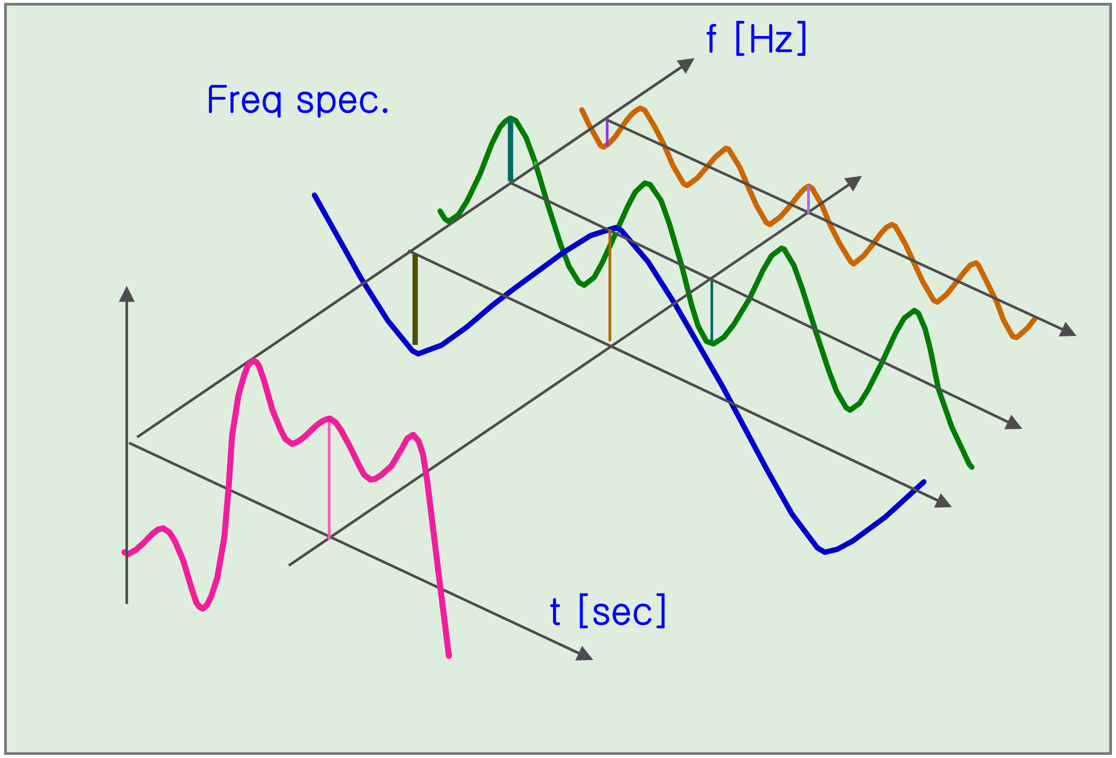
3.1 Frequency Analysis Concept

- The history of Fourier Analysis

  - Bernoulli(1753)
    - Linear combination of normal mode for string
  - Lagrange(1759)
    - Criticize about Bernoulli's opinion
  - Fourier(1807)
    - Harmonically Related Sinusoid Series for all periodic function.
  - Dirichlet(1829)
    - Made a mathematical base for Fourier Series
  - Cooley & Turkey(1967)
    - Fast Fourier Transform

- Fourier Series

$$x(t) = a_0 + a_1 \cos(\omega t + \theta_1) + \cdots + a_n \cos(n\omega t + \theta_n) + \cdots$$



- Fourier Series coefficients

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

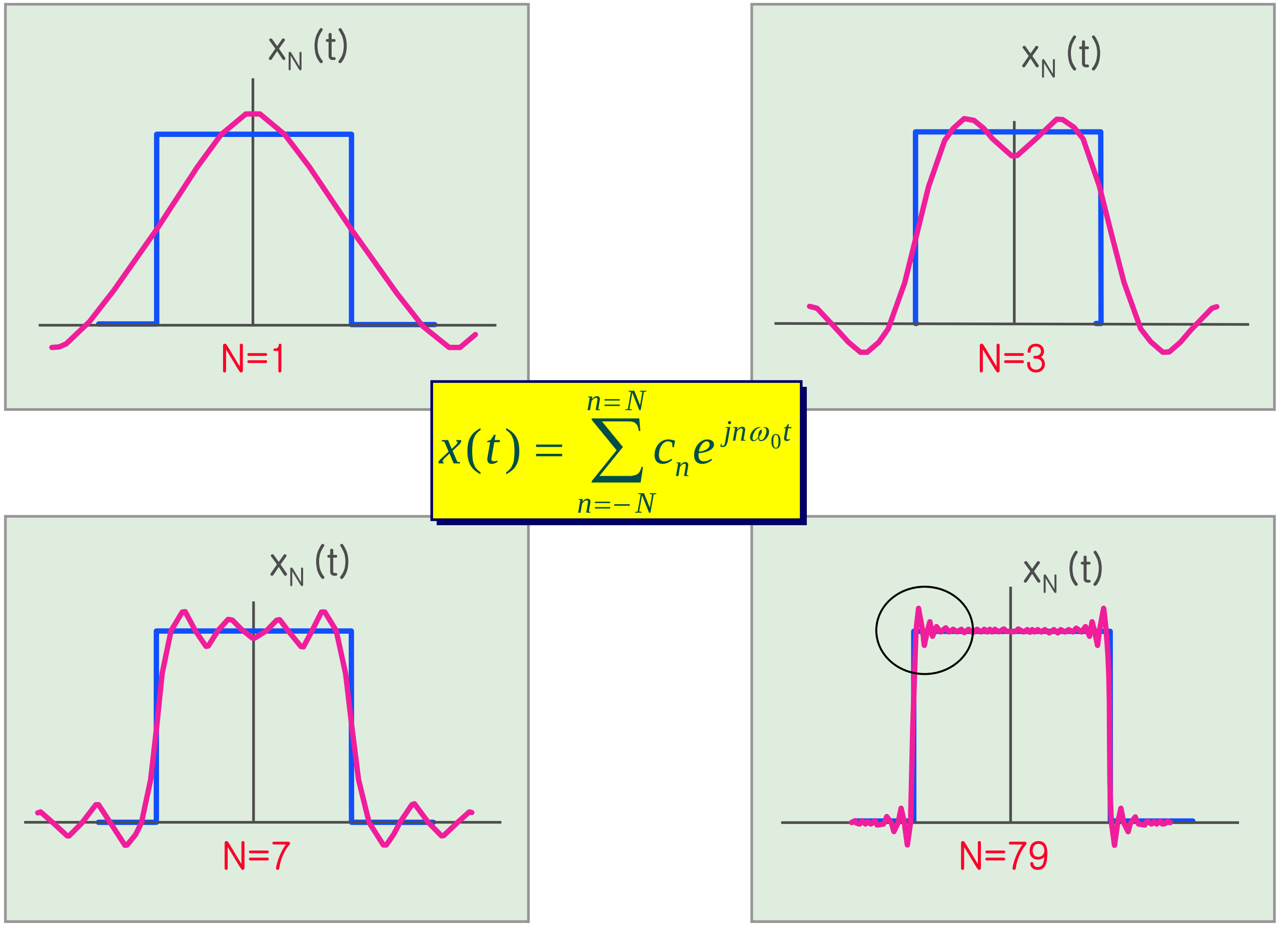
- x(t) : Periodic function

$$a_n = \frac{\int_0^T x(t)\cos n\omega_0 t dt}{\int_0^T \cos^2 n\omega_0 t dt} = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t dt$$

$$b_n = \frac{\int_0^T x(t)\sin n\omega_0 t dt}{\int_0^T \sin^2 n\omega_0 t dt} = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t dt$$

$$a_0 = \int_0^T x(t) dt \Big/ \int_0^T 1^2 dt = \frac{1}{T}\int_0^T x(t) dt$$

- Gibbs Phenomenon



$$x(t) = \sum_{n=-N}^{n=N} c_n e^{jn\omega_0 t}$$

- It always makes overshoot at discontinuous points.

- Complex Fourier Series

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

- x(t) : Periodic function

$$= a_0 + \sum_{n=1}^{\infty} \left\{ a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) + b_n \left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right) \right\}$$

$$= a_0 + \sum_{n=1}^{\infty} \left\{ \left(\frac{(a_n - jb_n)}{2} e^{jn\omega_0 t} \right) + \left(\frac{(a_n + jb_n)}{2} e^{-jn\omega_0 t} \right) \right\}$$

$$= c_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} c_{-n} e^{-jn\omega_0 t} = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_0 = a_0 \qquad c_n = \frac{a_n - jb_n}{2}$$

$$c_n = \frac{a_n - jb_n}{2} = \frac{1}{T}\int_0^T x(t)(\cos n\omega t - j\sin n\omega t)dt$$

$$= \frac{1}{T}\int_0^T x(t)e^{-j\omega t}dt$$

- Time Domain

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

⇄

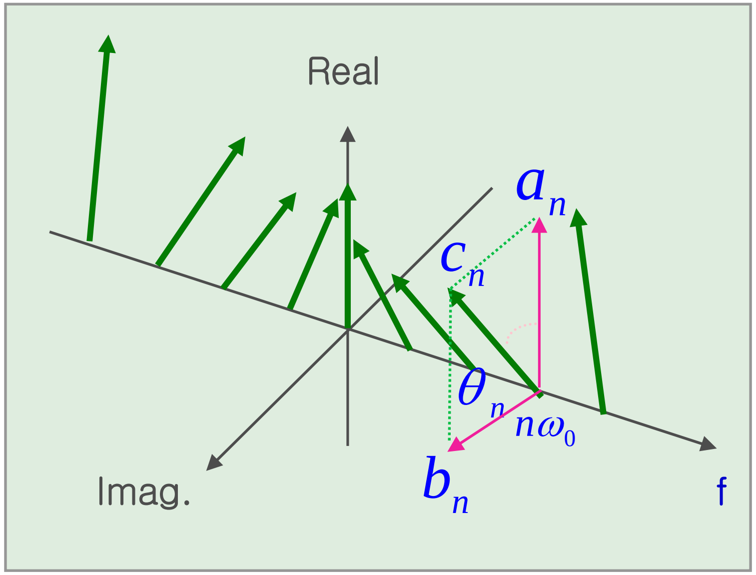
- Frequency Domain

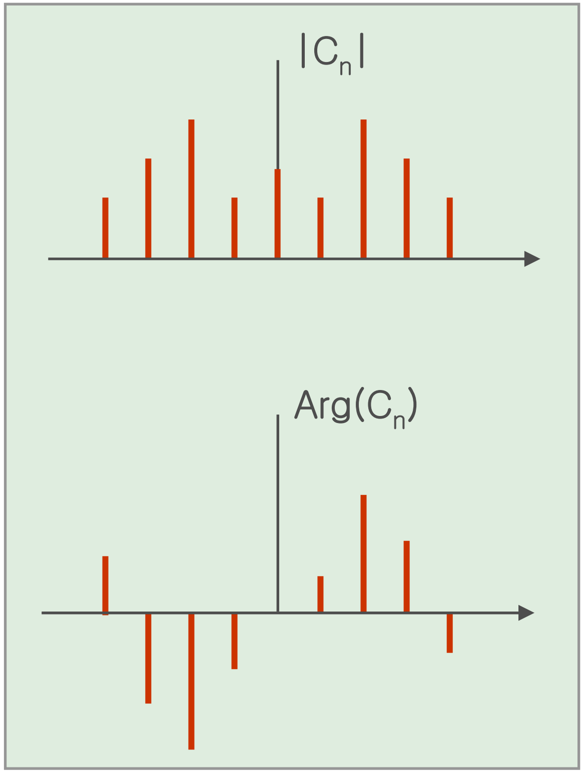
$$c_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt$$

- Fourier Spectrum

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt$$





- Fourier Transformation

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jn\omega_0 t}dt \qquad n\omega_0 = \frac{2\pi n}{T} = 2\pi\frac{n}{T}$$

If x(t) is non-periodic function, T→∞

$$\frac{n}{T} = n\Delta f \quad \rightarrow \quad f \qquad \therefore \quad \frac{2\pi n}{T} = 2\pi f = \omega$$

$$\lim_{T\to\infty}\left[c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jn\omega_0 t}dt\right] = 0 \quad \text{but} \quad \lim_{T\to\infty}\left[c_n T = \int_{-T/2}^{T/2} x(t)e^{-jn\omega_0 t}dt\right] = Cont.$$

$$\lim_{T\to\infty}\left[c_n T = \int_{-T/2}^{T/2} x(t)e^{-jn\omega_0 t}dt\right] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = X(\omega)$$

Fourier Transformation : $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = c_n T$

- Fourier Transformation

  - Frequency Domain

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

  - Time Domain

$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{+j\omega t} df$$

Proof) $x(t) = \int_{-\infty}^{\infty} X(\omega) e^{+j\omega t} df = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau} d\tau \; e^{j\omega t} df$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau} e^{j\omega t} d\tau \; df = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) e^{j\omega(t-\tau)} d\tau \; df$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) e^{j\omega(t-\tau)} d\tau \; df = \int_{-\infty}^{\infty} x(\tau) \int_{-\infty}^{\infty} e^{j\omega(t-\tau)} df \; d\tau$$

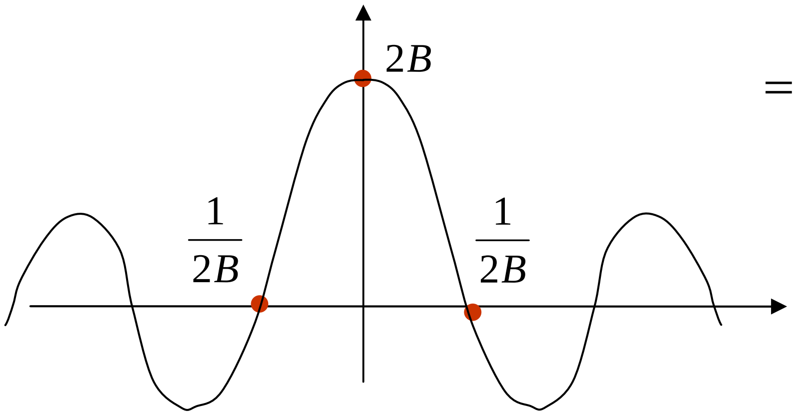
$$= \int_{-\infty}^{\infty} x(\tau) \, \delta(t-\tau) \, d\tau = x(t)$$

- Delta function

$$\begin{cases} \delta(\tau) = 0 & \tau \neq 0 \\ \delta(\tau) = \infty & \tau = 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(\tau)\,d\tau = 1 \quad \tau \neq 0$$

$$\int_{-\infty}^{\infty} \delta(\tau)\Phi(\tau)\,d\tau = \Phi(0)$$

$$\delta(\tau) = \int_{-\infty}^{\infty} e^{j2\pi f\tau}\,df = \lim_{B\to\infty} \int_{-B}^{B} e^{j2\pi f\tau}\,df = \lim_{B\to\infty} \frac{e^{j2\pi B\tau} - e^{-j2\pi B\tau}}{j2\pi\tau}$$



$$= \lim_{B\to\infty} \frac{\sin 2\pi B\tau}{\pi\tau}$$

Ex) Fourier transform of white noise

- Discrete Fourier Transformation

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} df \qquad x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} dt$$

$$\int \rightarrow \Sigma \qquad dt \rightarrow \triangle t = h$$

$$f \rightarrow f_k = k \triangle f \qquad t \rightarrow t_n = nh$$

$$X(f_k) = \sum_{n=0}^{N-1} x(nh) \exp[-j2\pi f_k \cdot nh] h \qquad x(nh) = \sum_{n=0}^{N-1} X(f_k) \exp[j2\pi f_k \cdot nh] \triangle f$$

$$\triangle f = \frac{1}{T_r} = \frac{1}{Nh} \qquad x(nh) = x(n) \qquad X(f_k) = X(k)$$

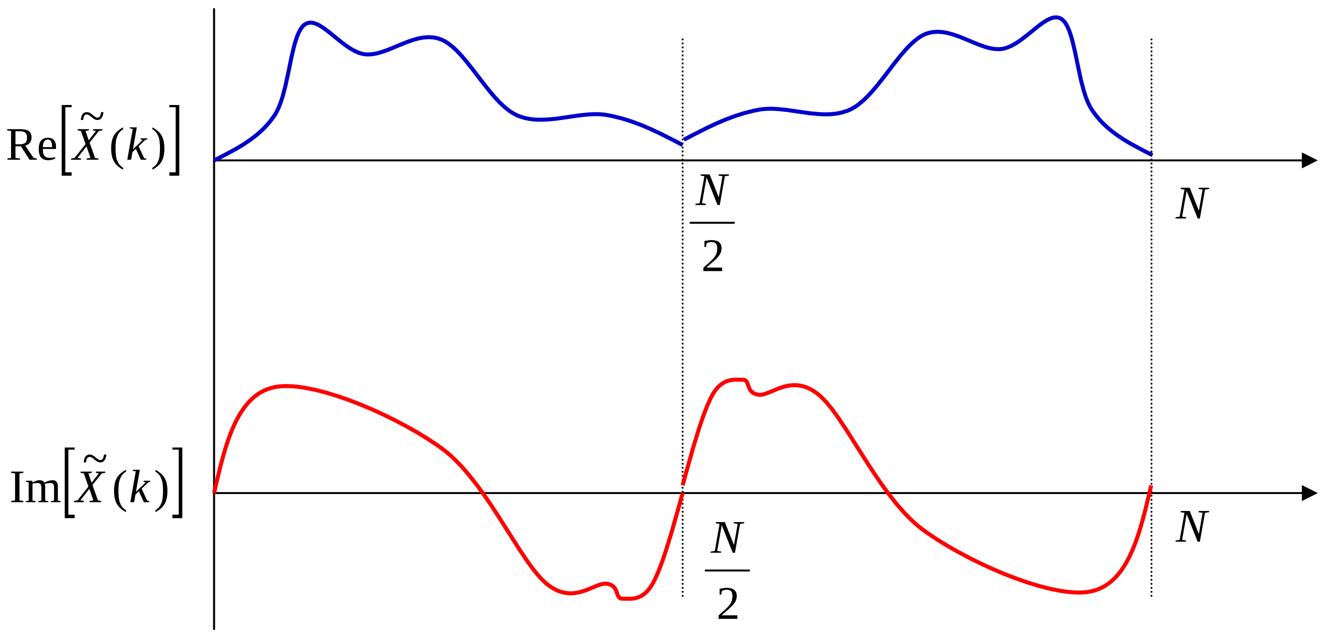
$$X(k) = \sum_{n=0}^{N-1} x(n) \exp\left[-j2\pi \frac{k}{Nh} \cdot nh\right] h \qquad x(n) = \sum_{k=0}^{N-1} X(n) \exp\left[j2\pi \frac{k}{Nh} \cdot nh\right] h$$

$$= h \cdot \sum_{n=0}^{N-1} x(n) \exp\left[-j2\pi \frac{kn}{N}\right] \qquad = \frac{1}{Nh} \cdot \sum_{k=0}^{N-1} x(n) \exp\left[j2\pi \frac{kn}{N}\right]$$

- Discrete Fourier Transformation

$$\text{Let} \quad \tilde{X}(k) = \frac{1}{h} X(k),$$

$$\tilde{X}(k) = \sum_{n=0}^{N-1} x(n) \exp\left[-j2\frac{kn}{N}\right], \qquad x(n) = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{X}(k) \exp\left[j2\pi\frac{kn}{N}\right]$$



- Parseval's theorem

Average power of x(t) = power of individual frequency component

$$x^2(t) = x(t)x^*(t) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} c_n e^{jn\omega t} c_m^* e^{-jn\omega t}$$

$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} c_n c_m^* e^{j(n-m)\omega t}$$

Average power

$$= \frac{1}{T}\int_0^T x^2(t)dt$$

$$= \frac{1}{T}\int_0^T \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} c_n c_m^* e^{j(n-m)\omega t} dt = \sum_{n=-\infty}^{\infty} \left| c_n \right|^2$$

$$n \neq m \quad \Rightarrow \quad \int_0^T e^{i(n-m)\omega t} dt = 0$$

- Power spectrum

Auto PSD : $S'_{xx}(f) = \dfrac{X^*(f)\bullet X(f)}{T}$

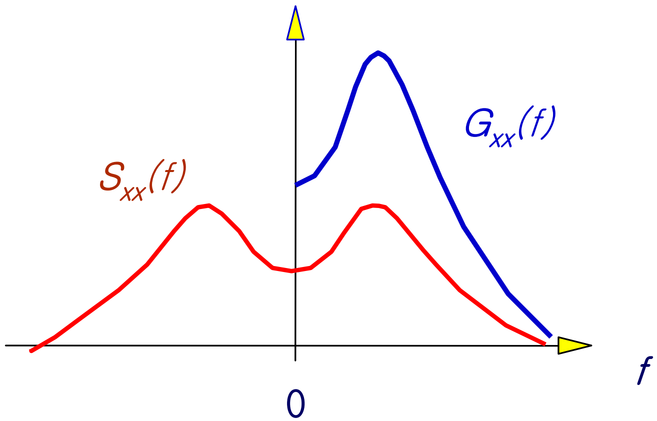
- Spectrum calculation of FFT

$$S_{xx}(f) = X^*(f)\bullet X(f)$$

: 2-sided spectrum

$$G_{xx}(f) = \begin{cases} 2S_{xx}(f), & f > 0 \\ S_{xx}(f), & f = 0 \\ 0, & f < 0 \end{cases}$$

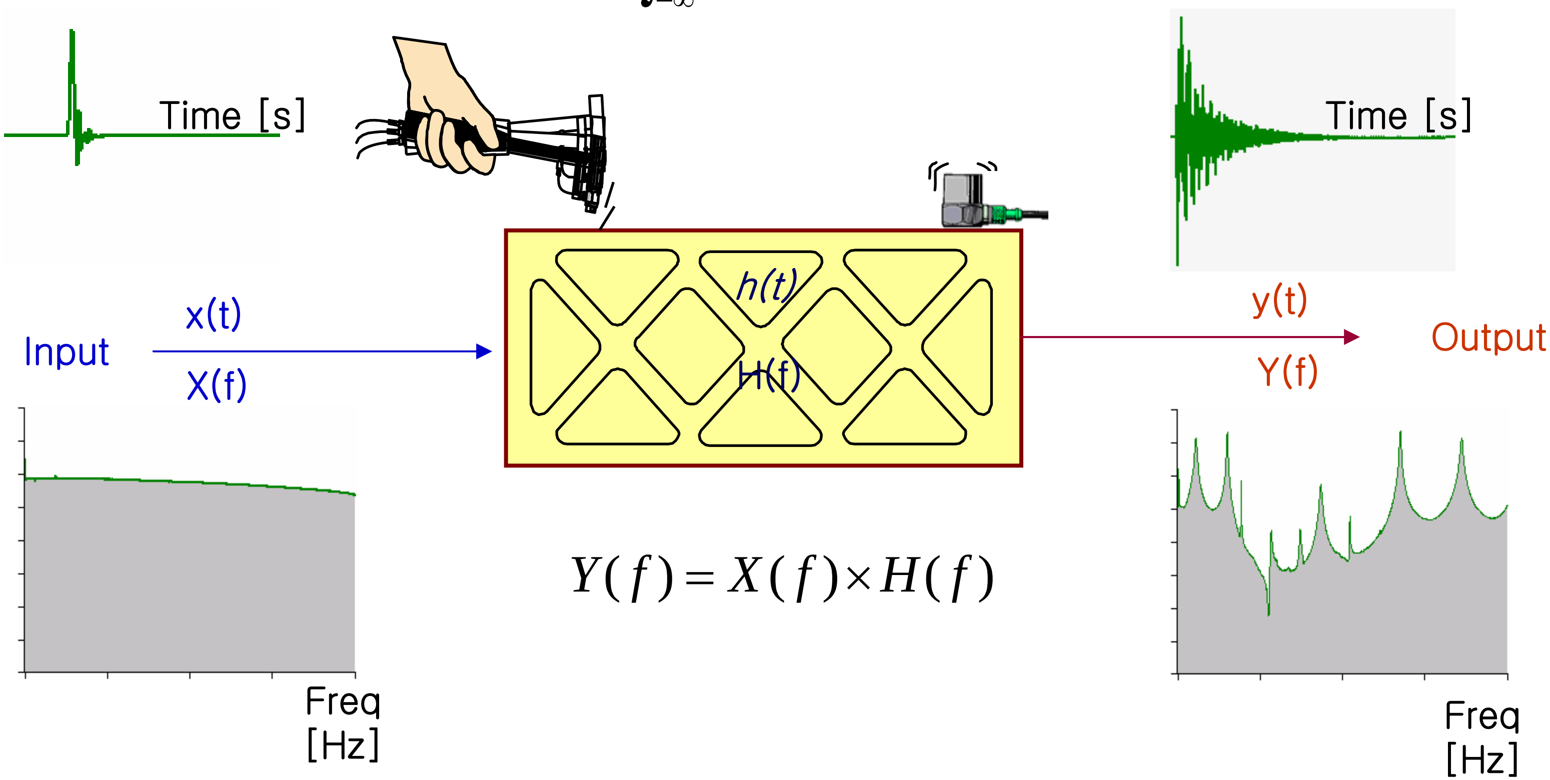
: 1-sided spectrum



- X(f) : (instantaneous) Fourier transform
- X*(f) : complex conjugate of X(f)

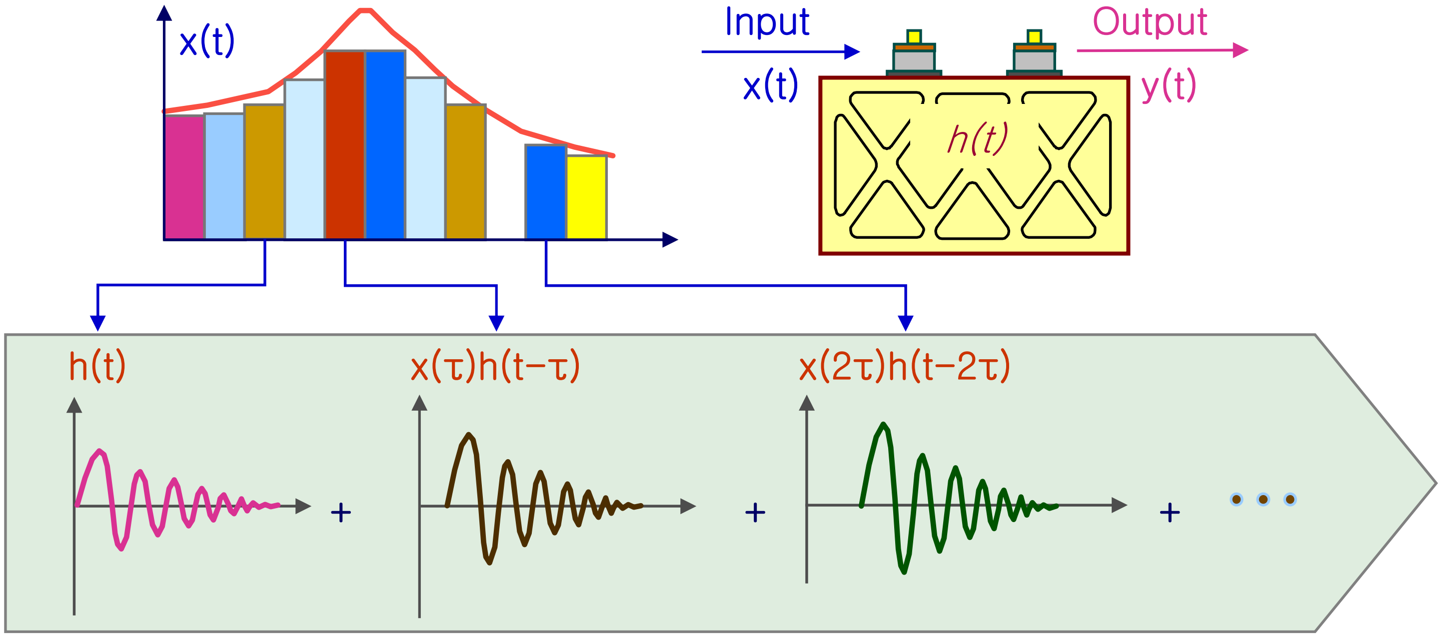
- Convolution & Multiplication

$$y(t)=\int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau$$



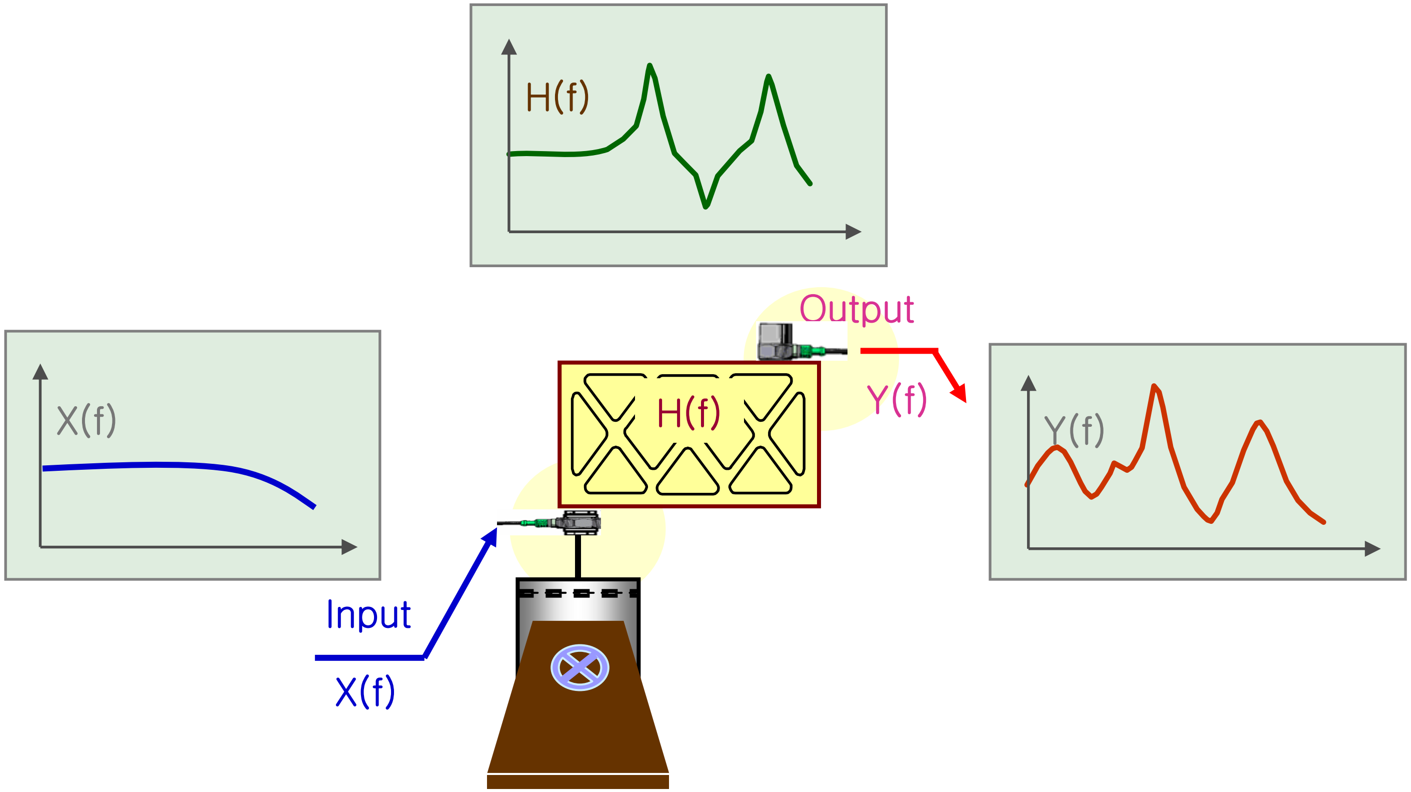
$$Y(f)=X(f)\times H(f)$$

- Time Domain : Convolution

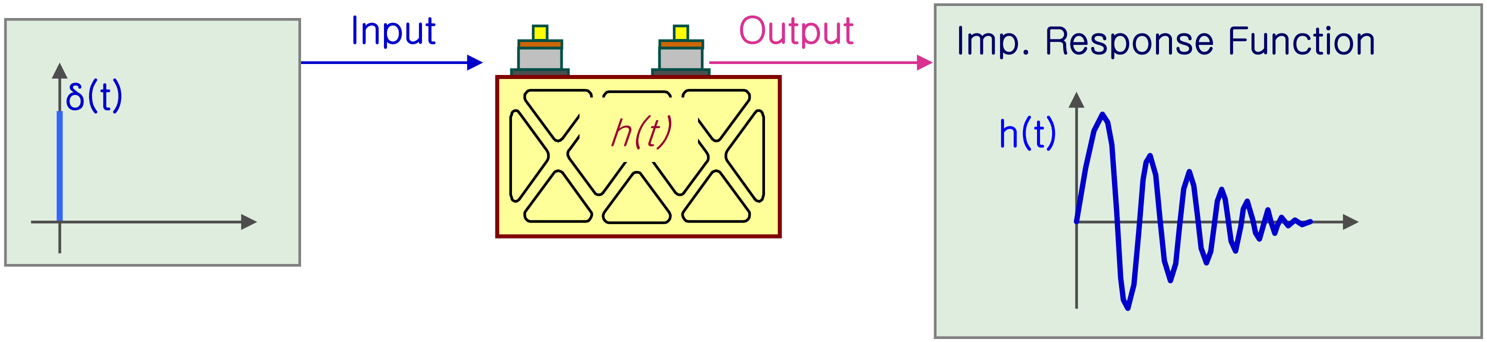


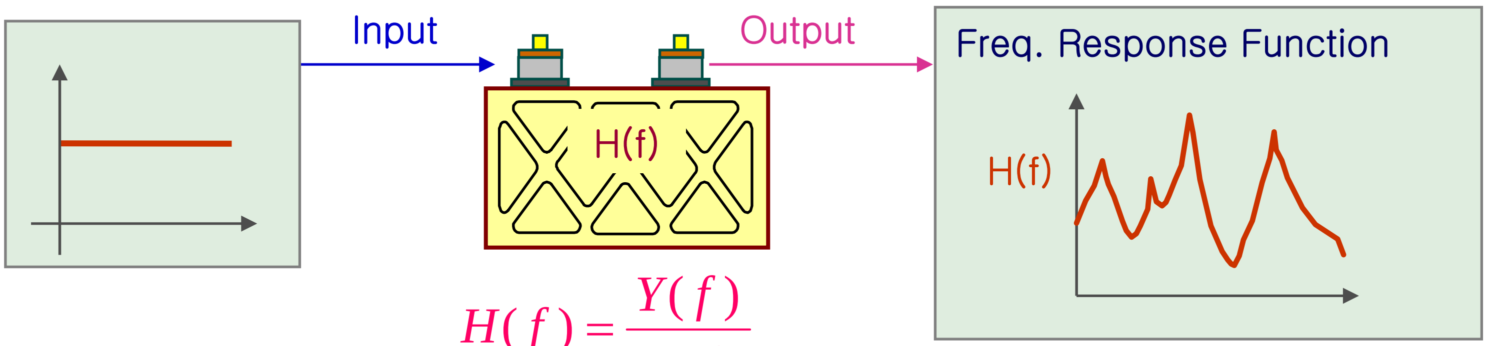
- Determine of System damping
- Measure Reverberation Time

- Frequency Domain : Multiplication



- System Analysis

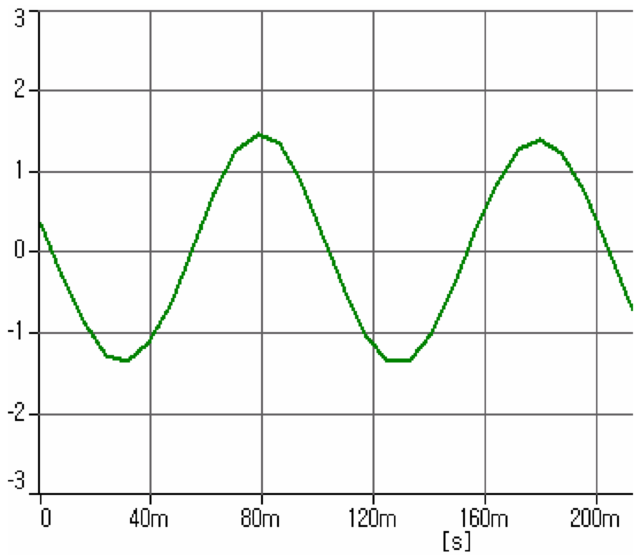




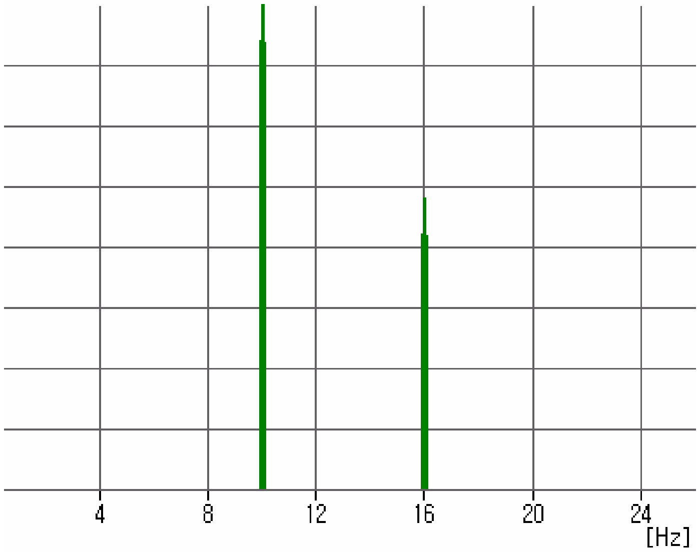
$$H(f) = \frac{Y(f)}{X(f)}$$

• Obscure Signal Separation

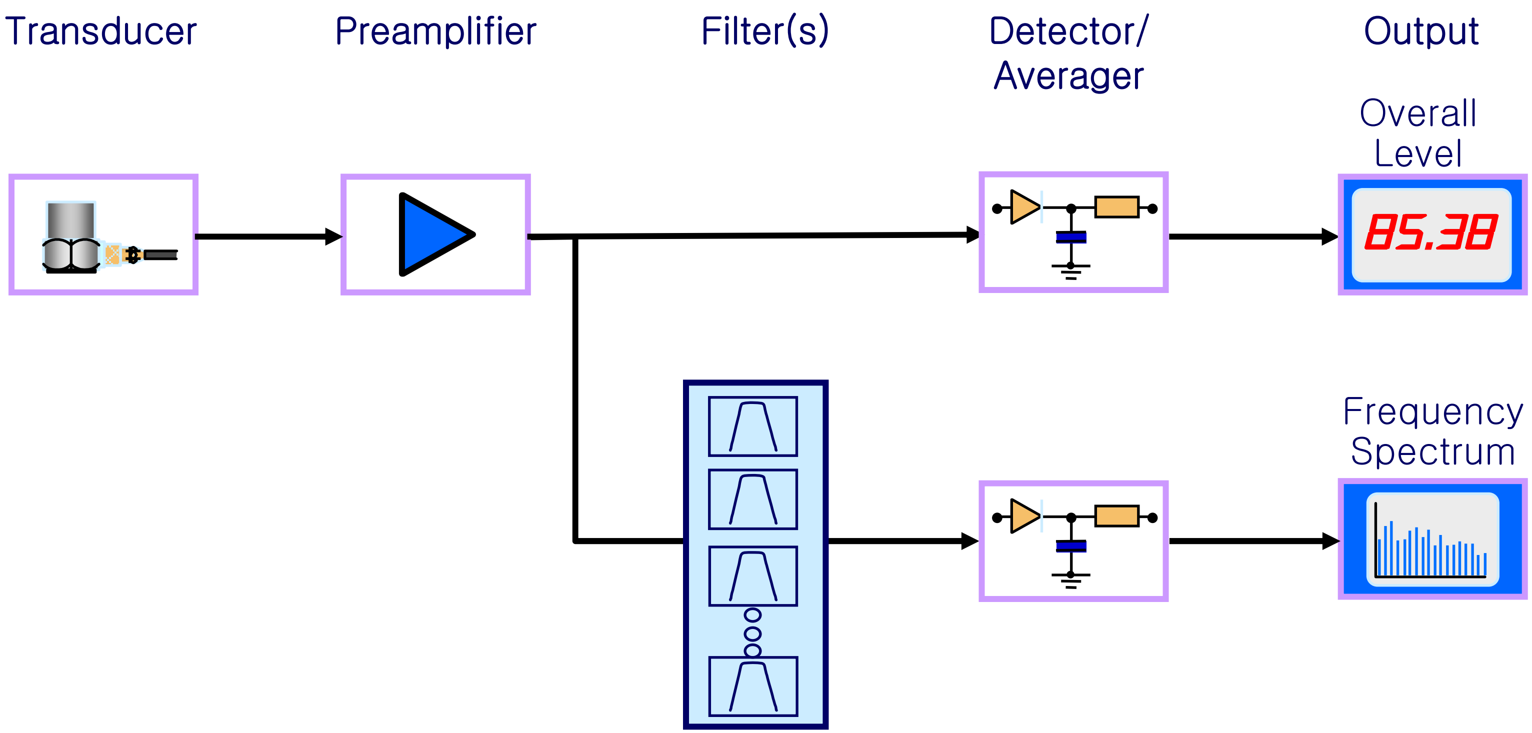
Time domain



Frequency domain

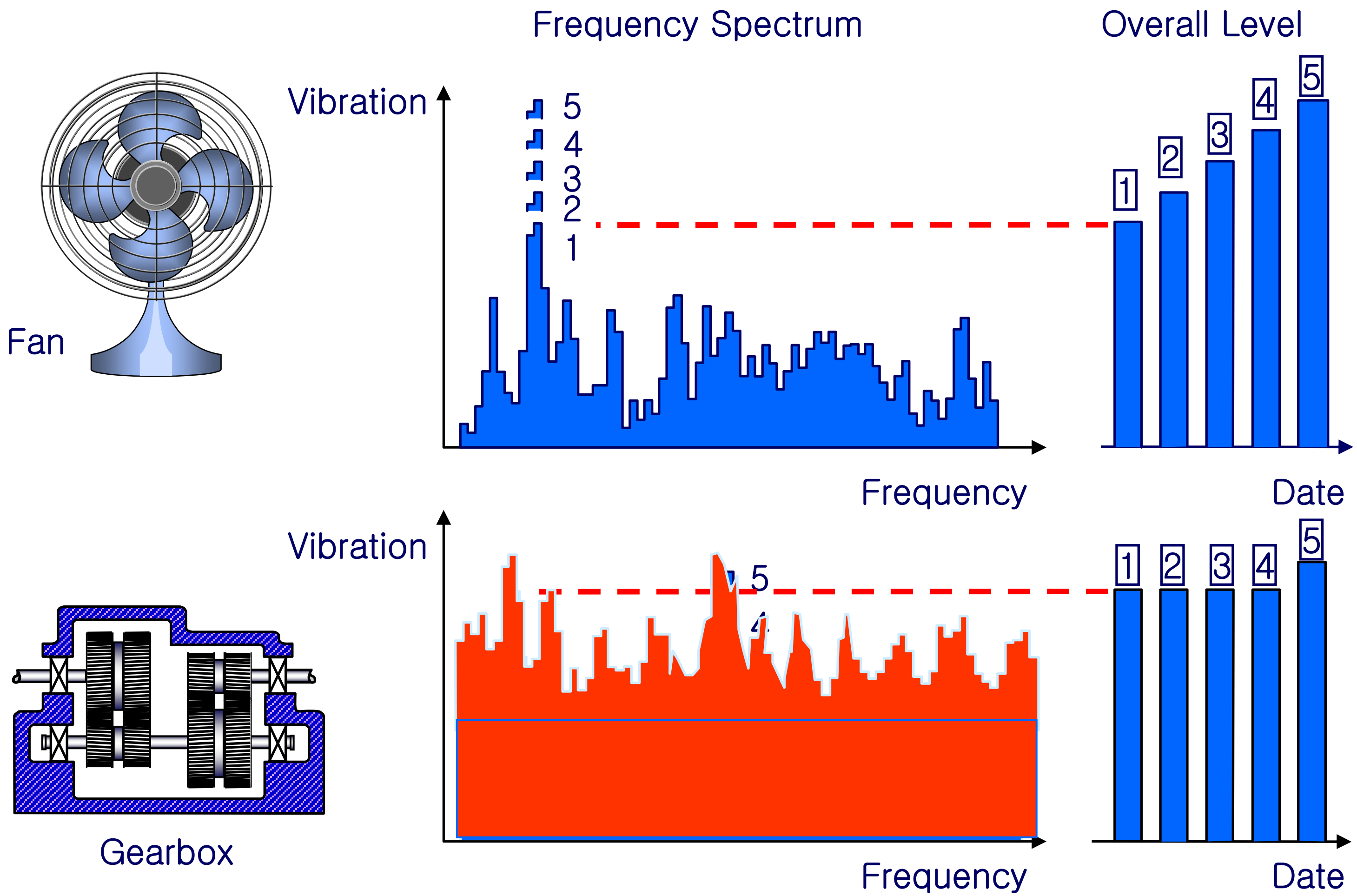


- Frequency Spectrum vs. Overall Level(I)



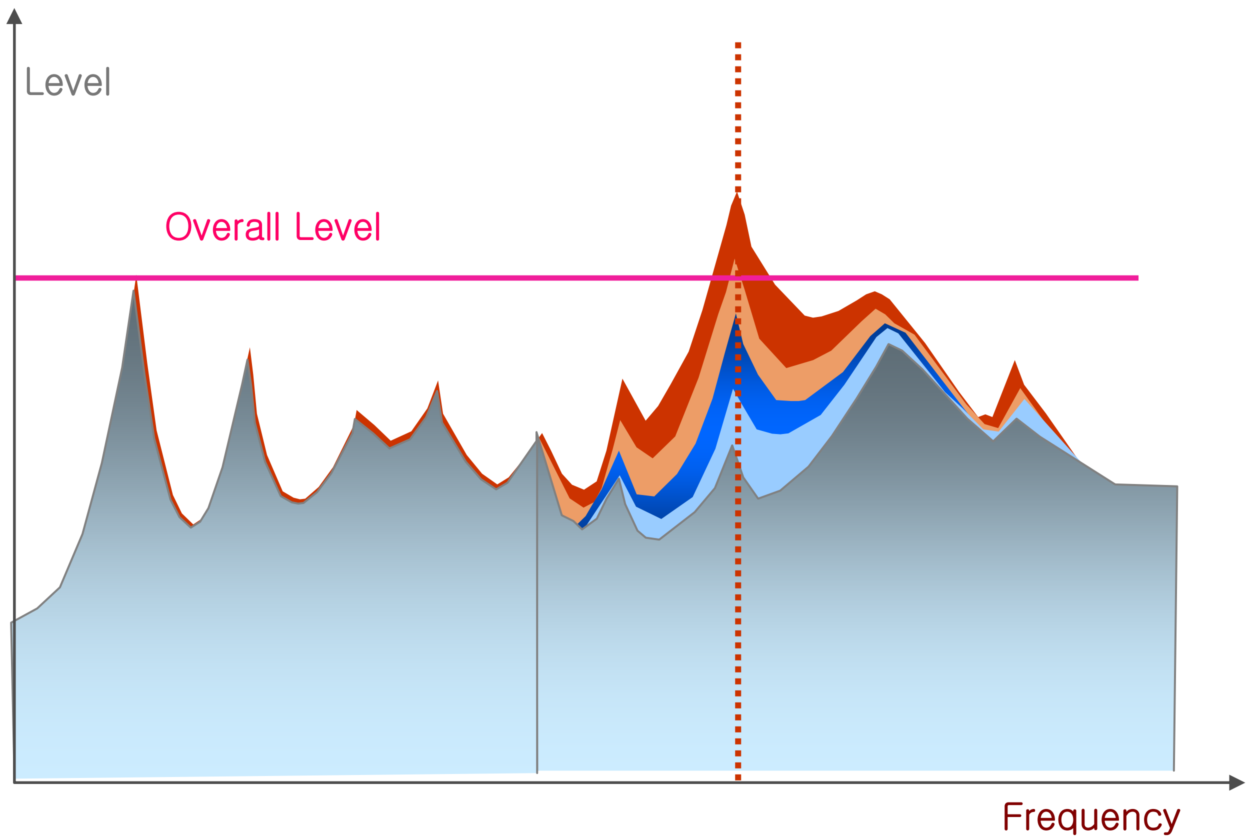
- The frequency spectrum gives in many cases a detailed information about the signal sources which cannot be obtained from the time signal.

- Frequency Spectrum vs. Overall Level (II)



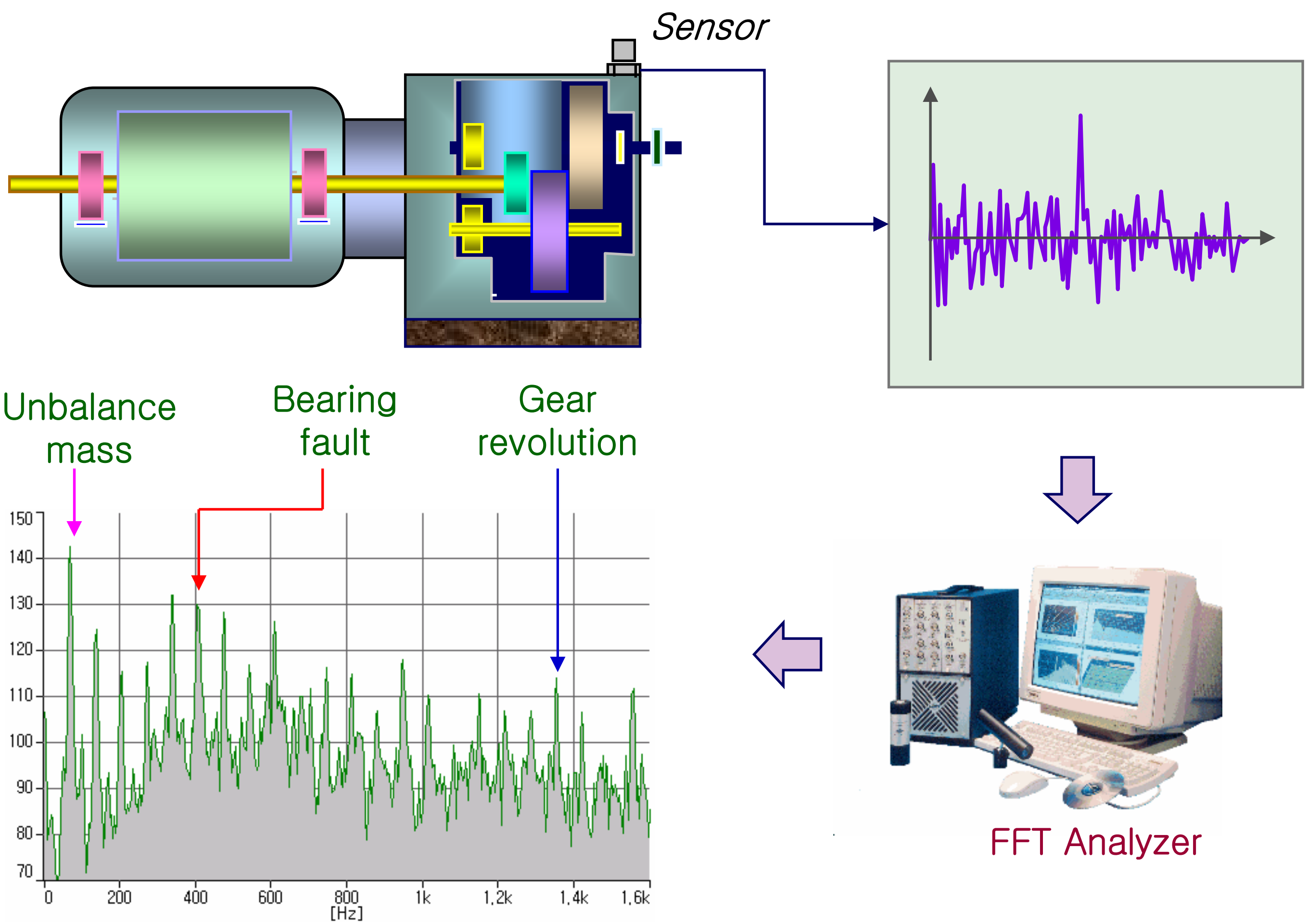
- A general rule is overall measurements are permissible for simple, non critical machines, while more complex, more critical machinery requires spectral analysis.

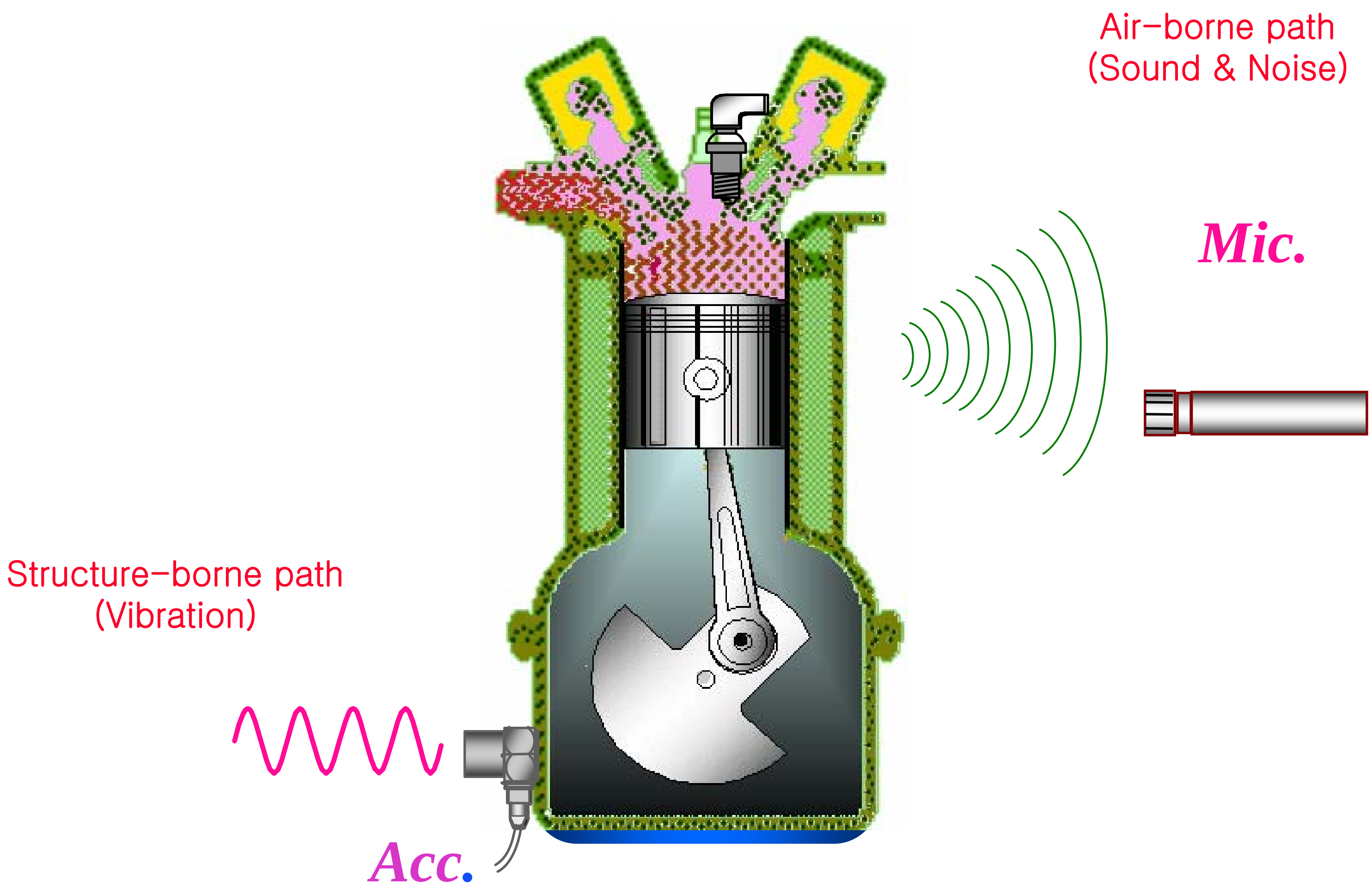
- Condition Diagnosis for Mechanical system



A frequency analysis of the vibration, on the other hand will give a much earlier warning of the fault, since it is selective, and will allow the increasing vibration at the frequency associated with the fault to be identified.

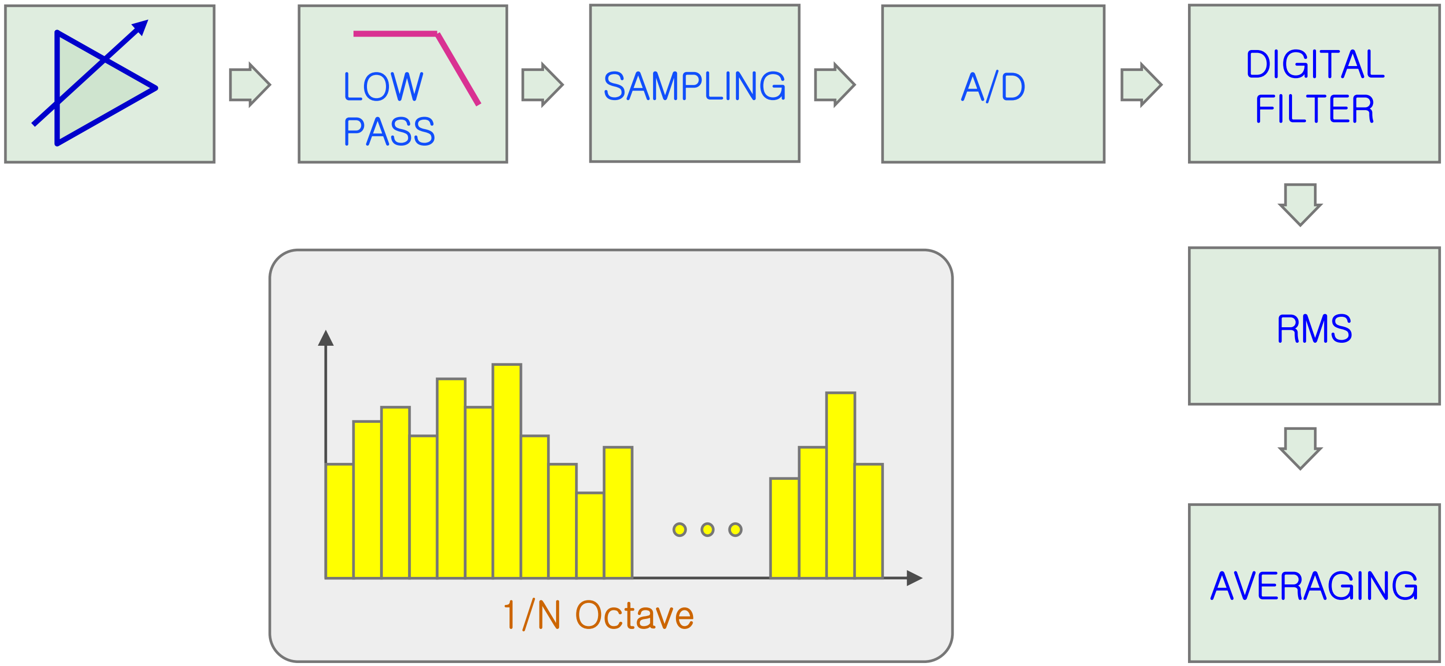
- Source Identification





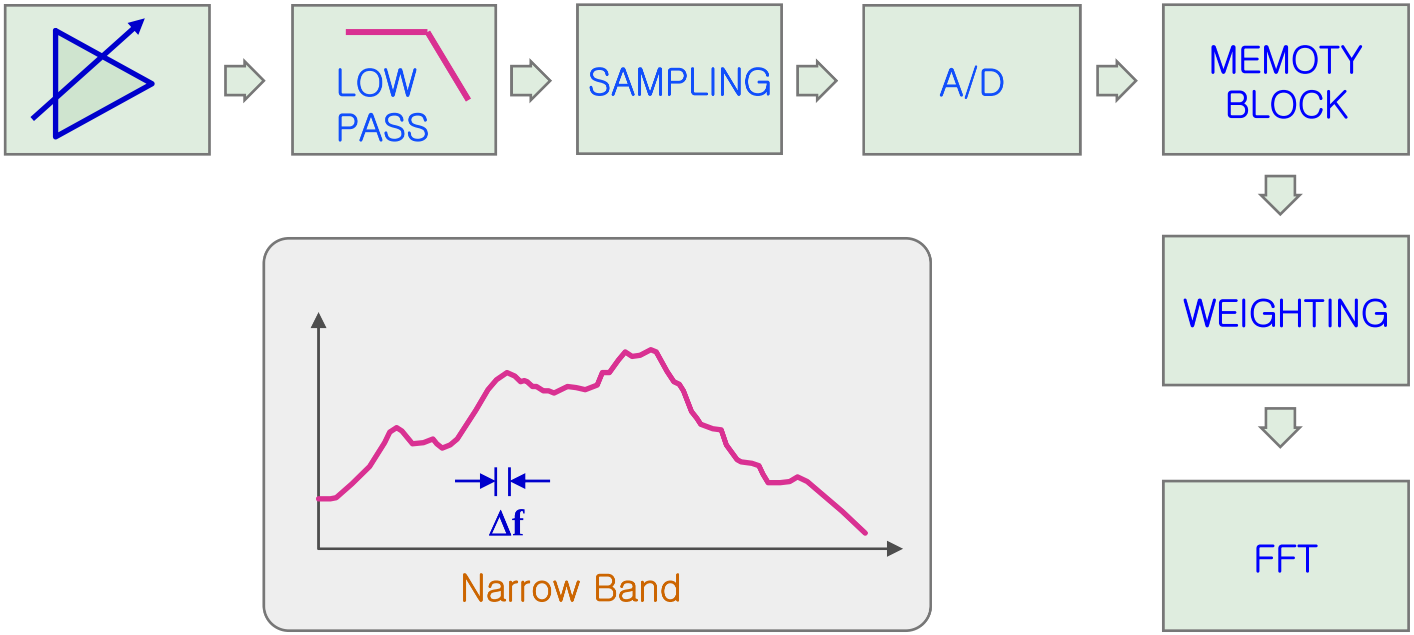
A frequency analysis of the vibration, on the other hand will give a much earlier warning of the fault, since it is selective, and will allow the increasing vibration at the frequency associated with the fault to be identified.

- Digital Filter



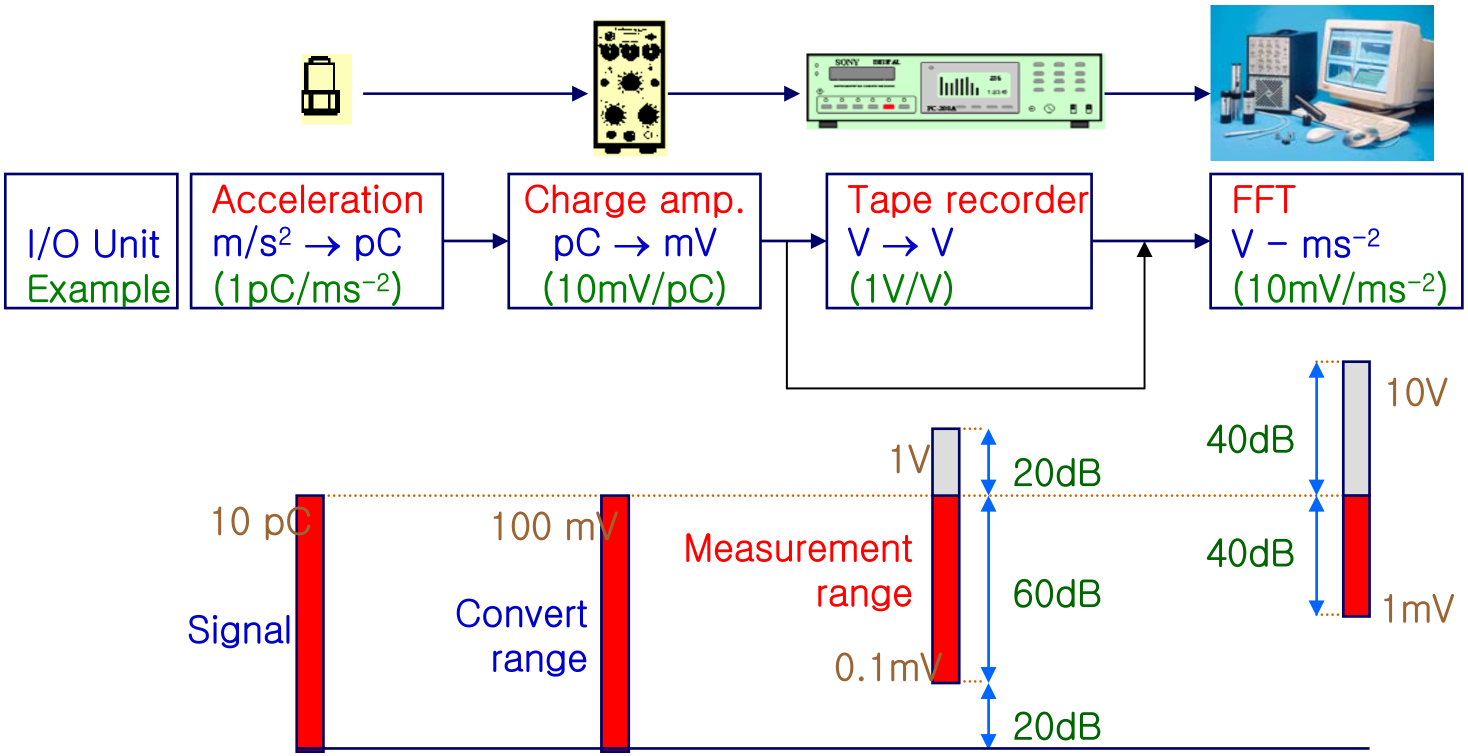
One type of digital analyzer is the Digital Filter Analyzer, which uses digital filters. Due to a very high calculation speed, the analyzer can operate in Real Time in the whole if its frequency range from 1.6Hz to 20 kHz.

- FFT

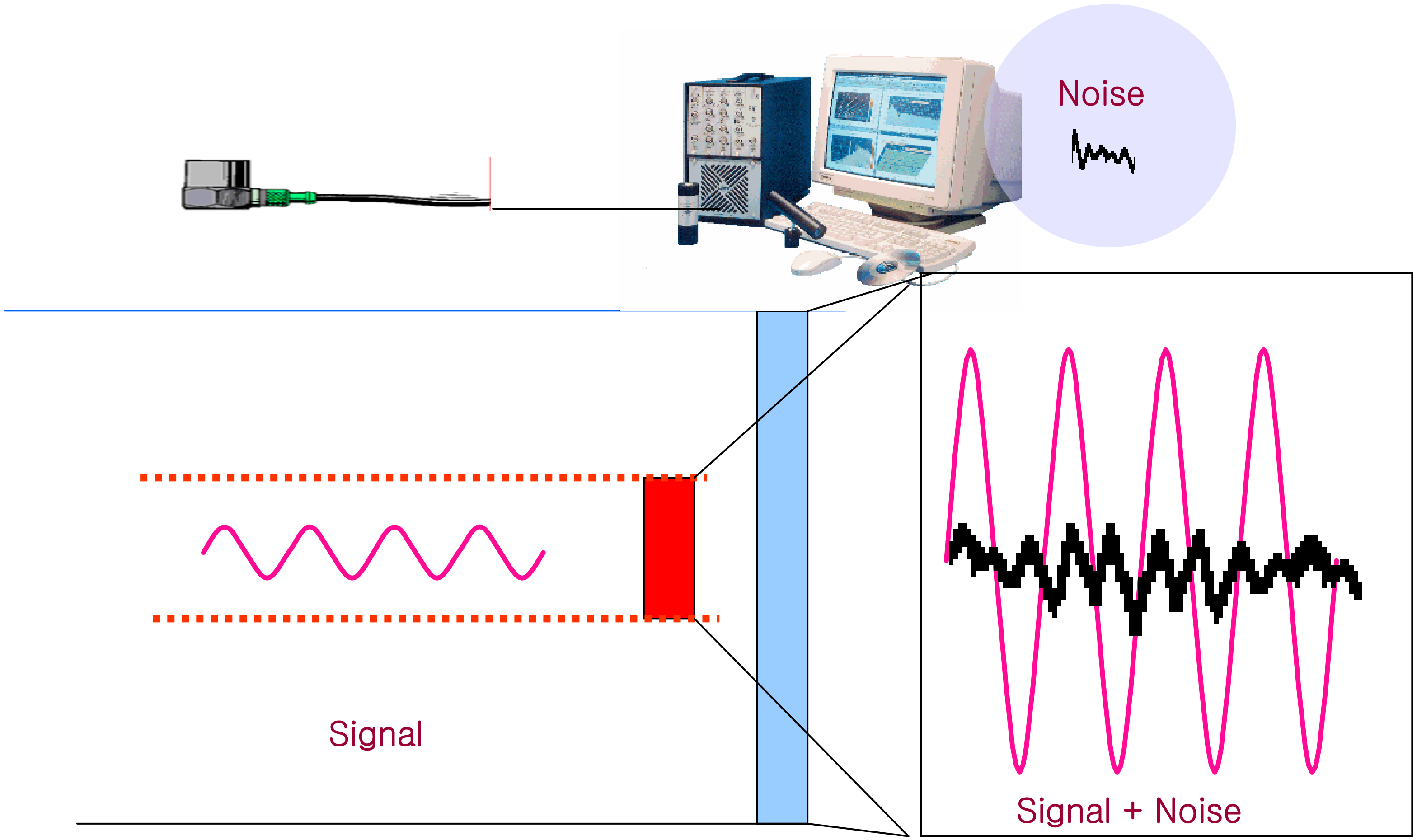


The FFT analyzer will take a block of data and make a calculation of the frequency content in this block of data, Both the time signal (the block of data) and the frequency spectrum can be displayed on a large screen

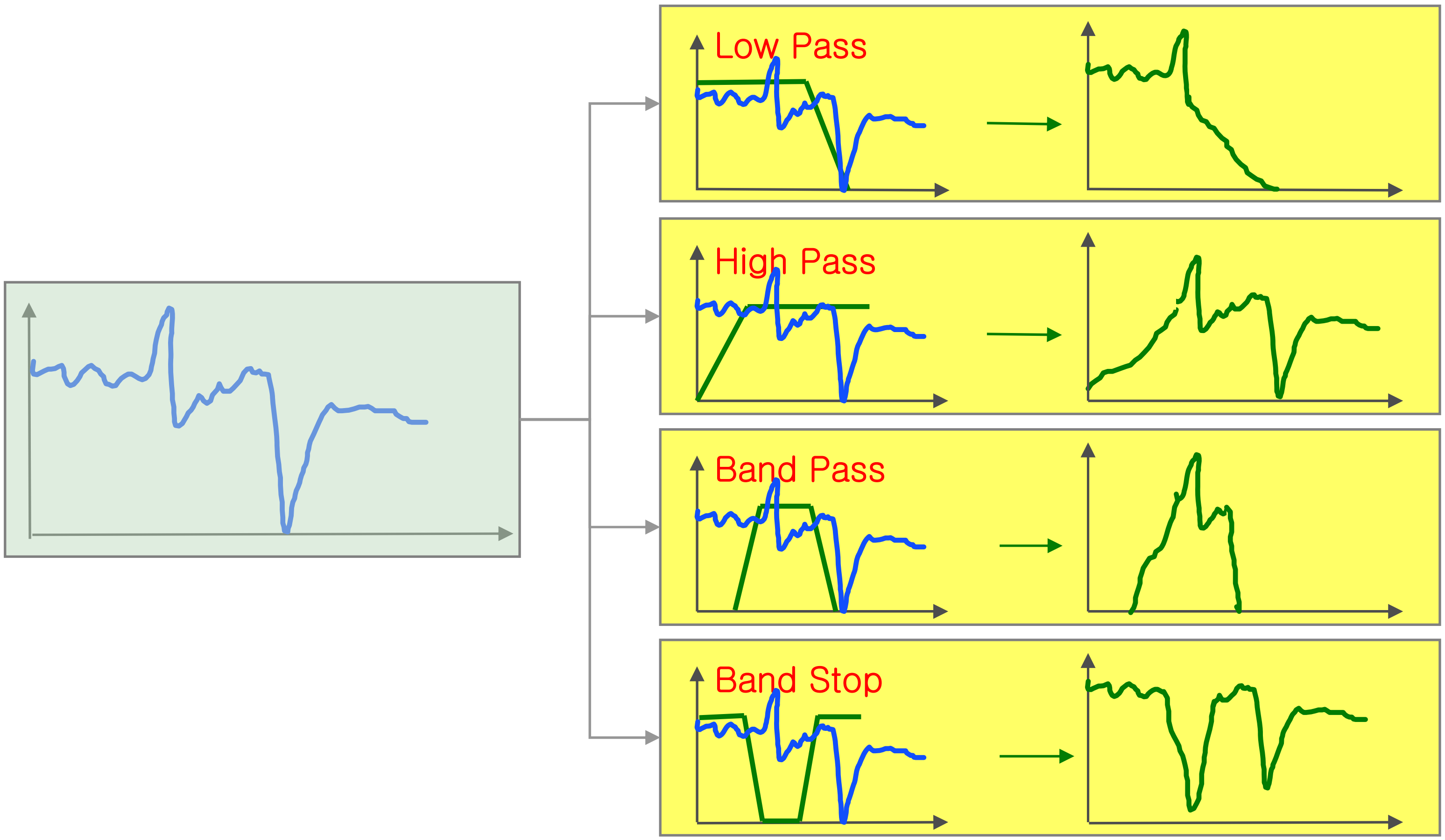
- Dynamic Range



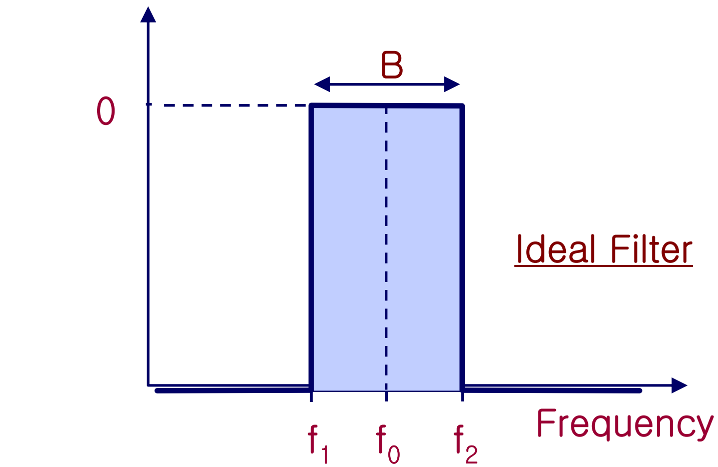
- Dynamic Range – Signal to Noise ratio



- Types of Filters

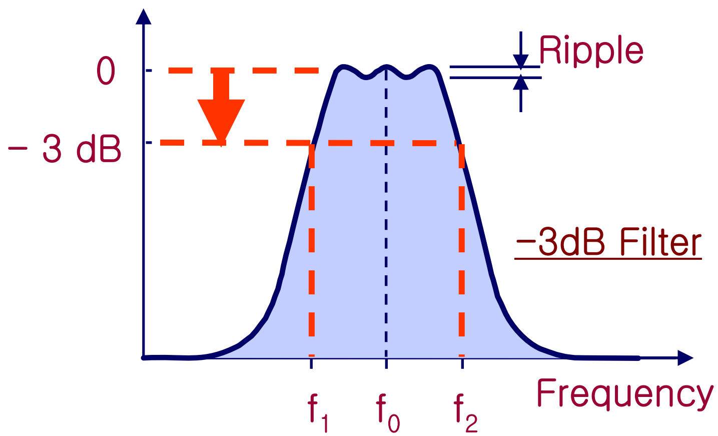


- Bandpass Filters and Bandwidth

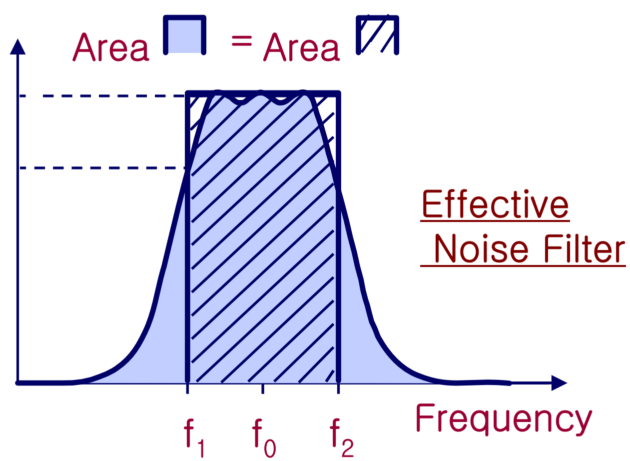


Bandwidth = $f_2 - f_1$

Centre Frequency = f_0

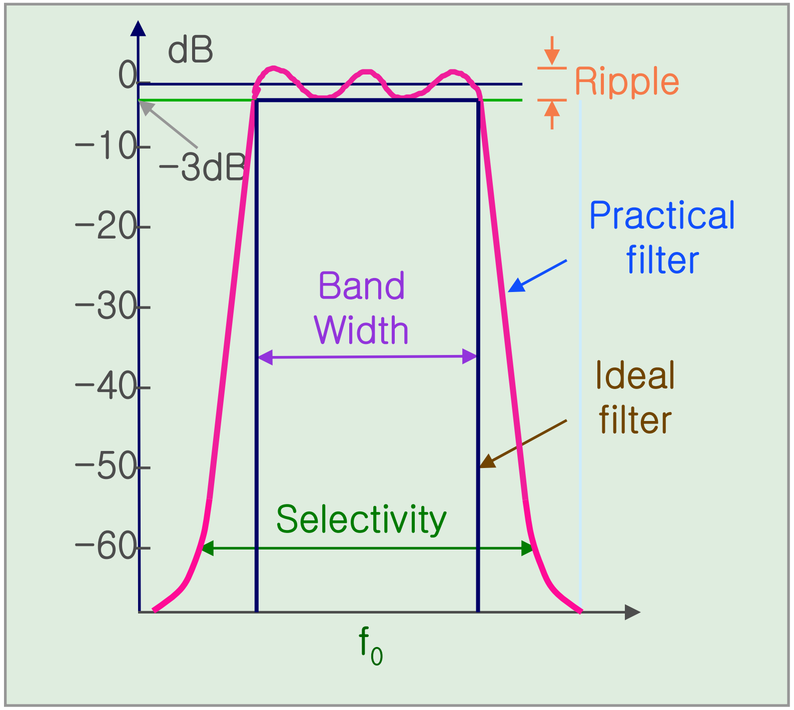


Y–axis : linear scale



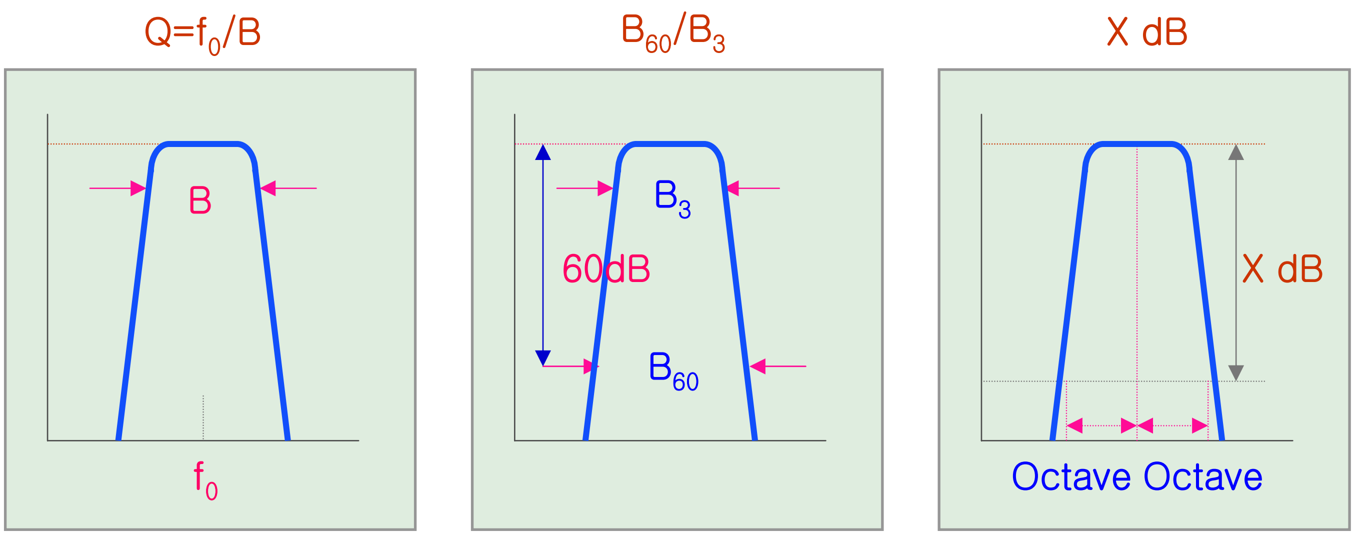
- Filter Characteristic

Y-axis : log scale



Center frequency : f_0
In band ripple
Bandwidth : B_3 or B_{noise}
Selectivity : Shape Factor $=B_{60}/B_3$

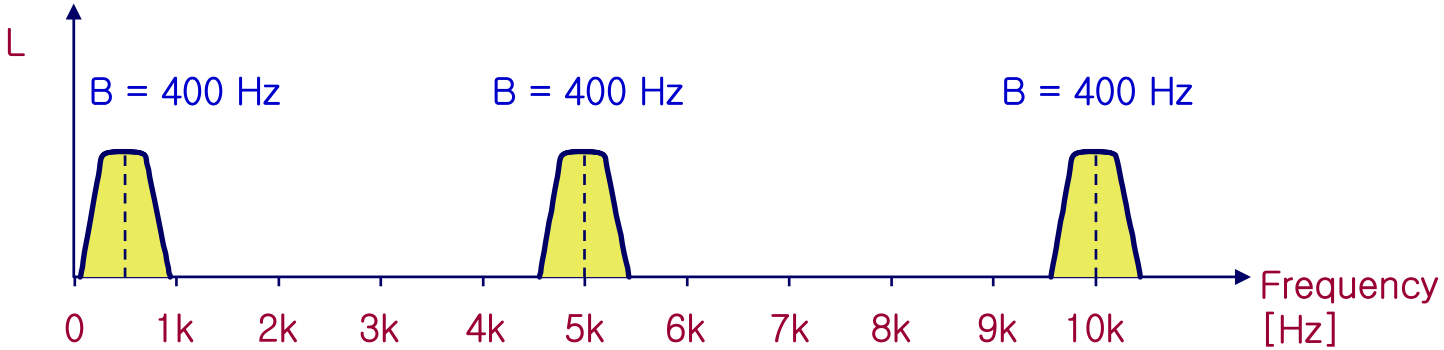
- How good is the Filter



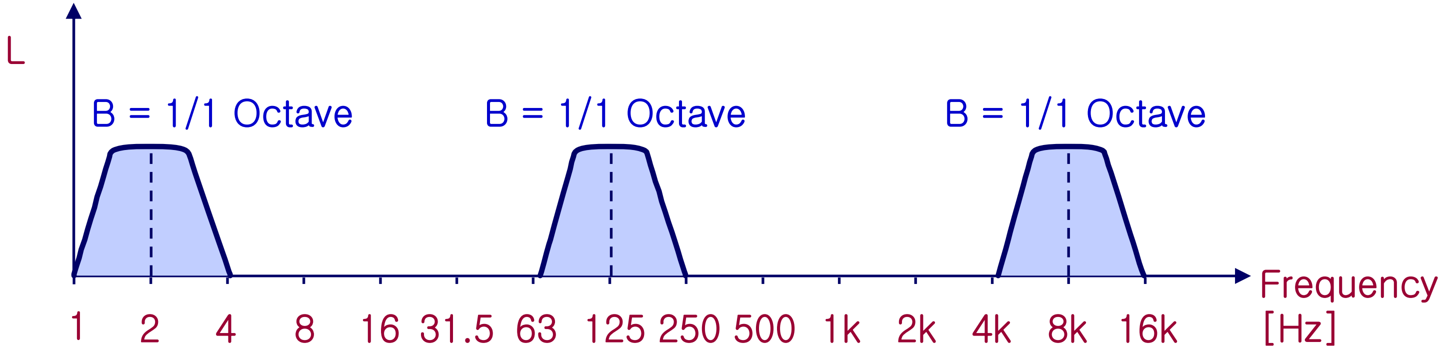
Quality factor — Shape factor — Octave selectivity

- Quality-factor(Q Factor). Sometimes used to specify how good a relative bandwidth filter is more often used to describe the response of mechanical resonating structures. These will actually act as band pass filters.
- Shape-Factor. Used to specify how good constant bandwidth filters are.
- Octave-selectivity. Used to specify how good relative bandwidth filters are. An octave corresponds to a factor of two on the frequency scale I.e. a doubling or a halving of the center frequency.

- Filter Types and Frequency Scales



Linear Frequency Axis
(primarily used in vibration analysis)

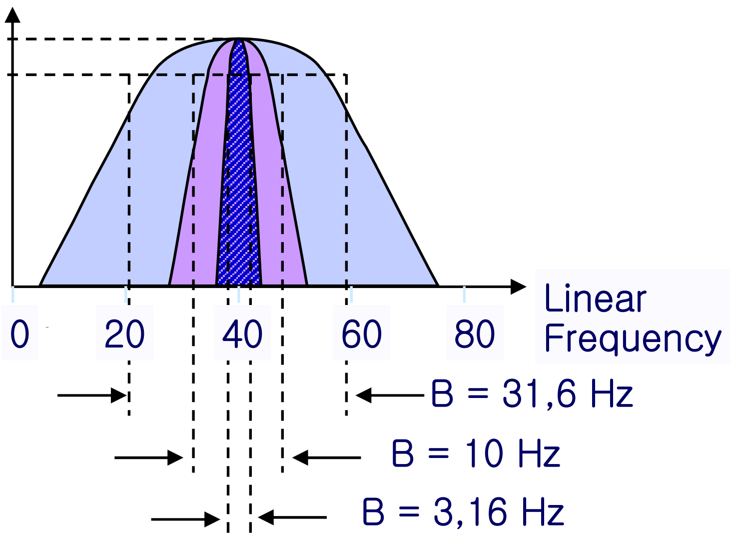


Logarithmic Frequency Axis
(primarily used in acoustic analysis)

- Types of Band-pass Filters

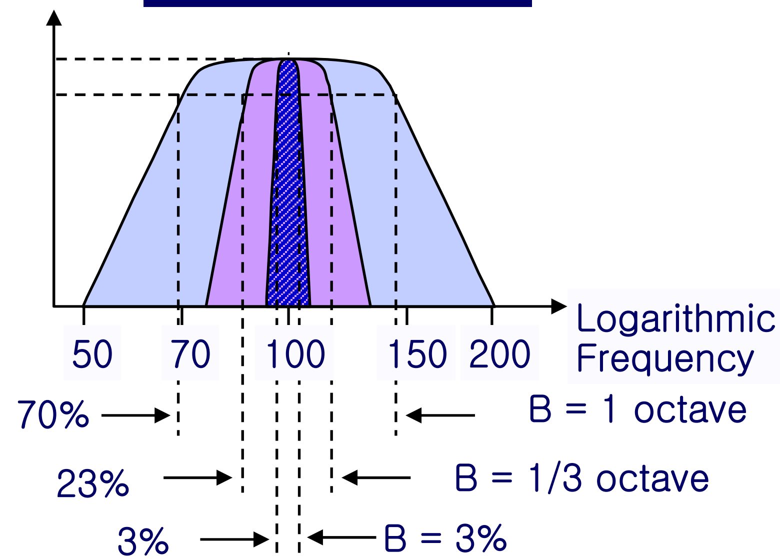
Constant Bandwidth

$$B = x \text{ Hz}$$

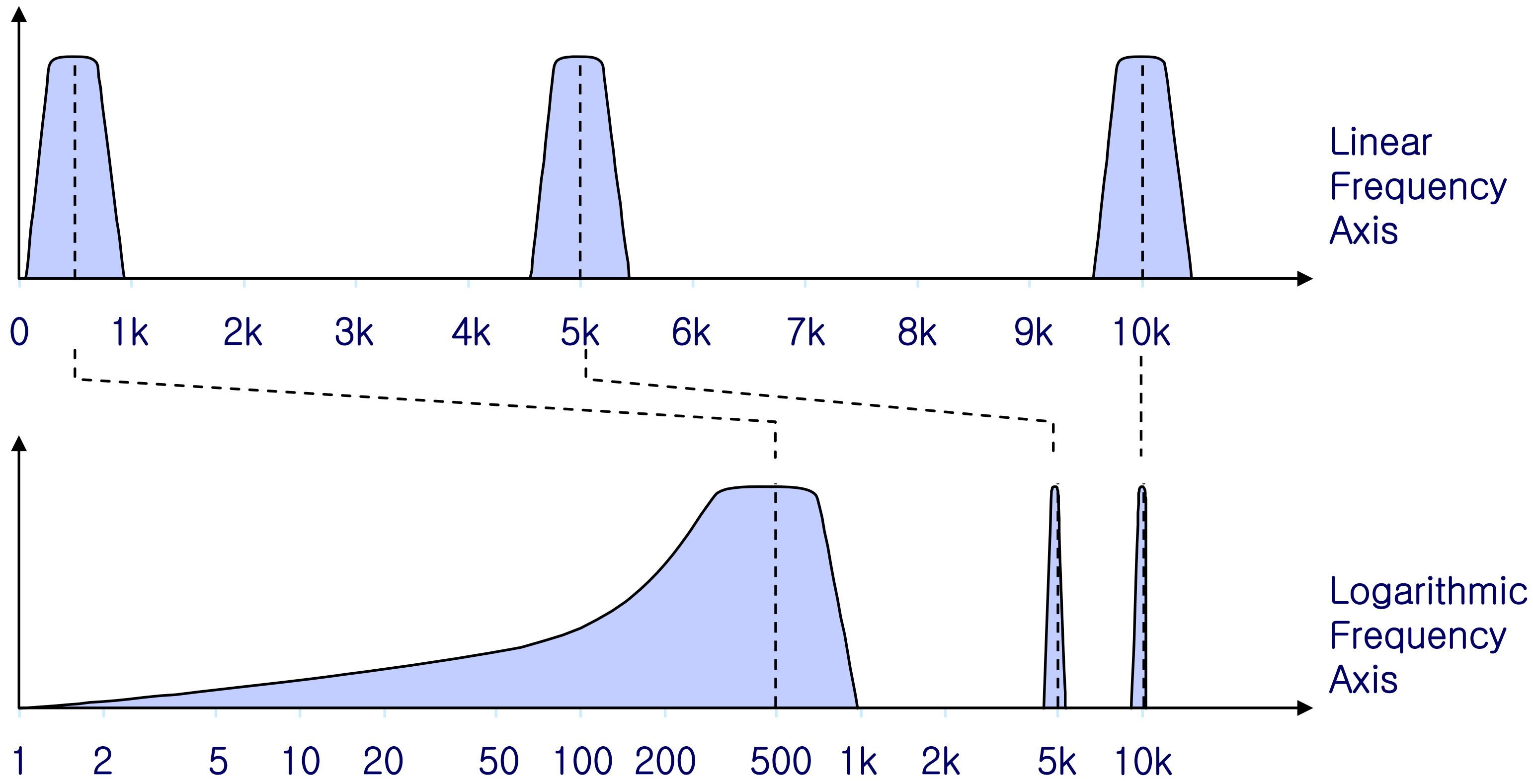


Constant Percentage Bandwidth
or Relative Bandwidth

$$B = y\% = \frac{y \,´\, f_0}{100}$$

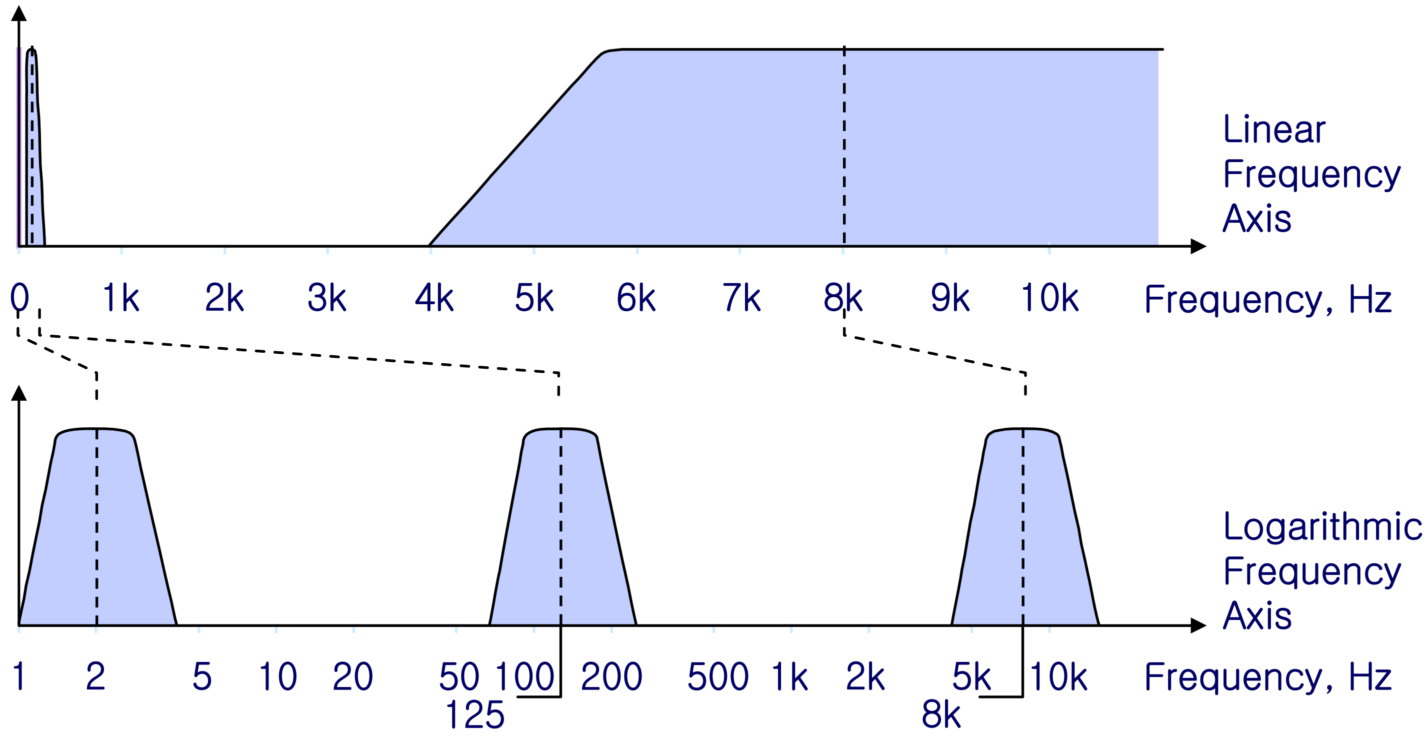


- Constant Bandwidth Filtering

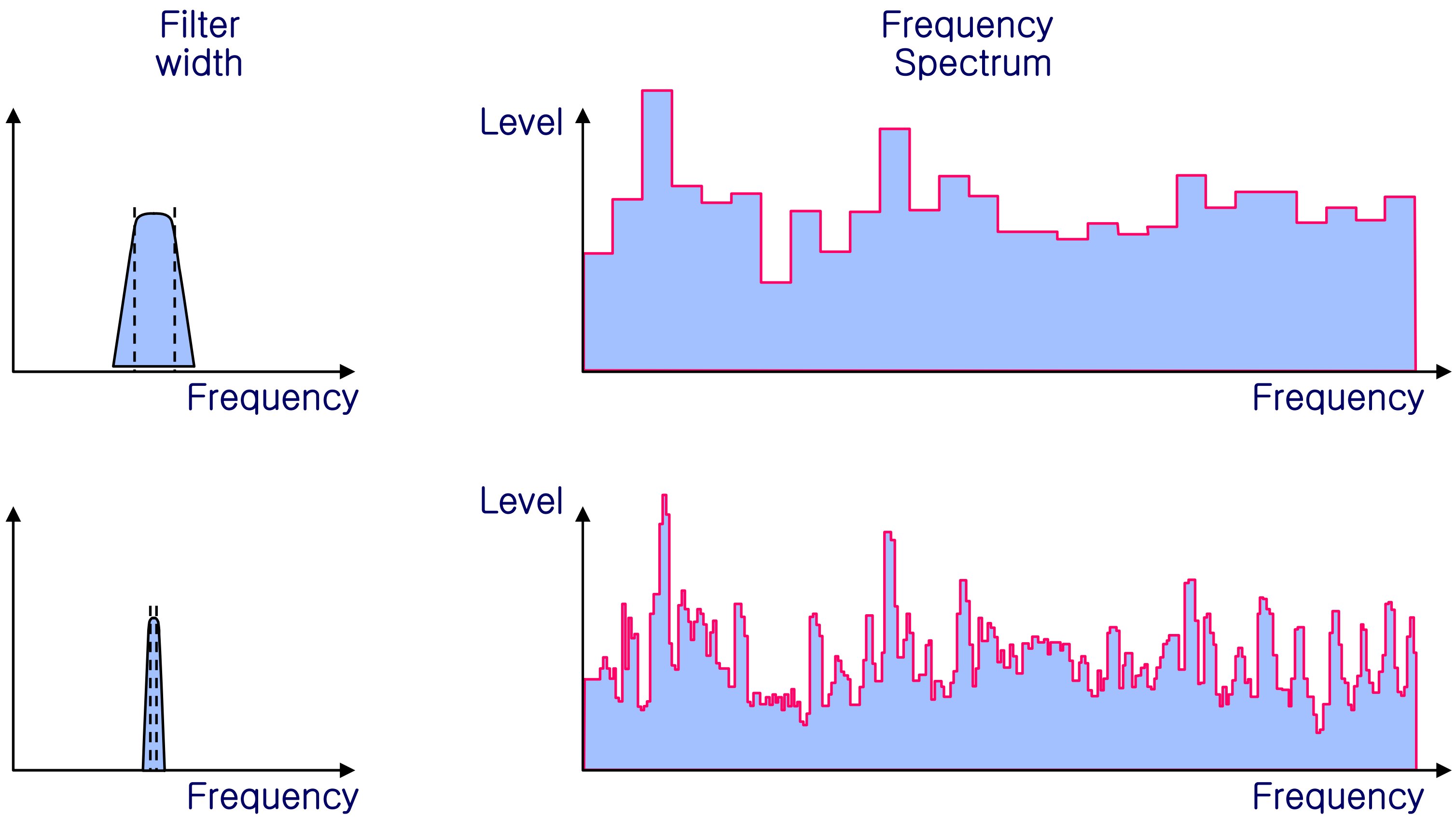


- Constant Percentage Bandwidth Filters

Bandwidth = 1/1 octave = 70% of Centre Frequency

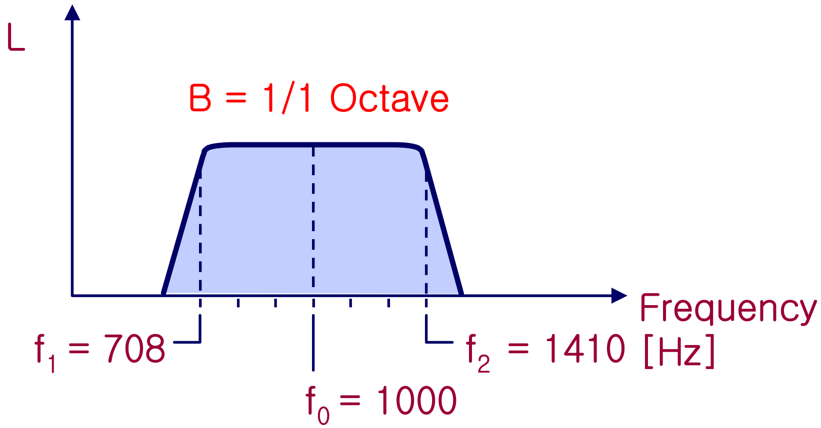


- Selecting Bandwidth



3.3 Measurement of Sound & Vibration

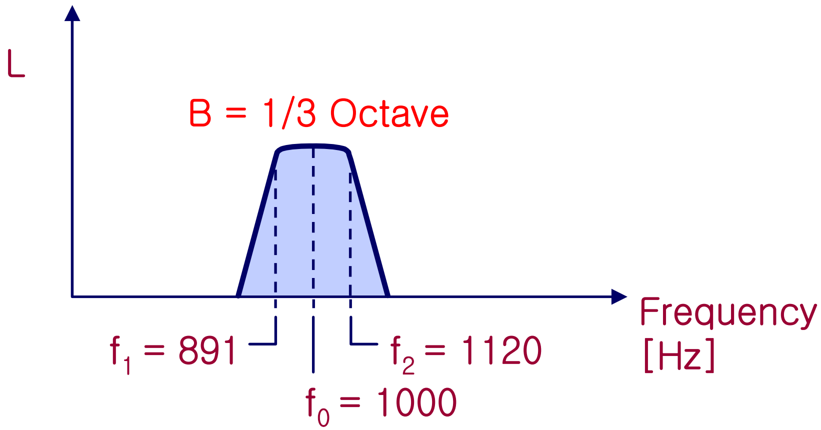
- 1/1 and 1/3 Octave Filters



1/1 Octave

$$f_2 = 2 \times f_1$$

$$B = 0.7 \times f_0 \approx 70\%$$

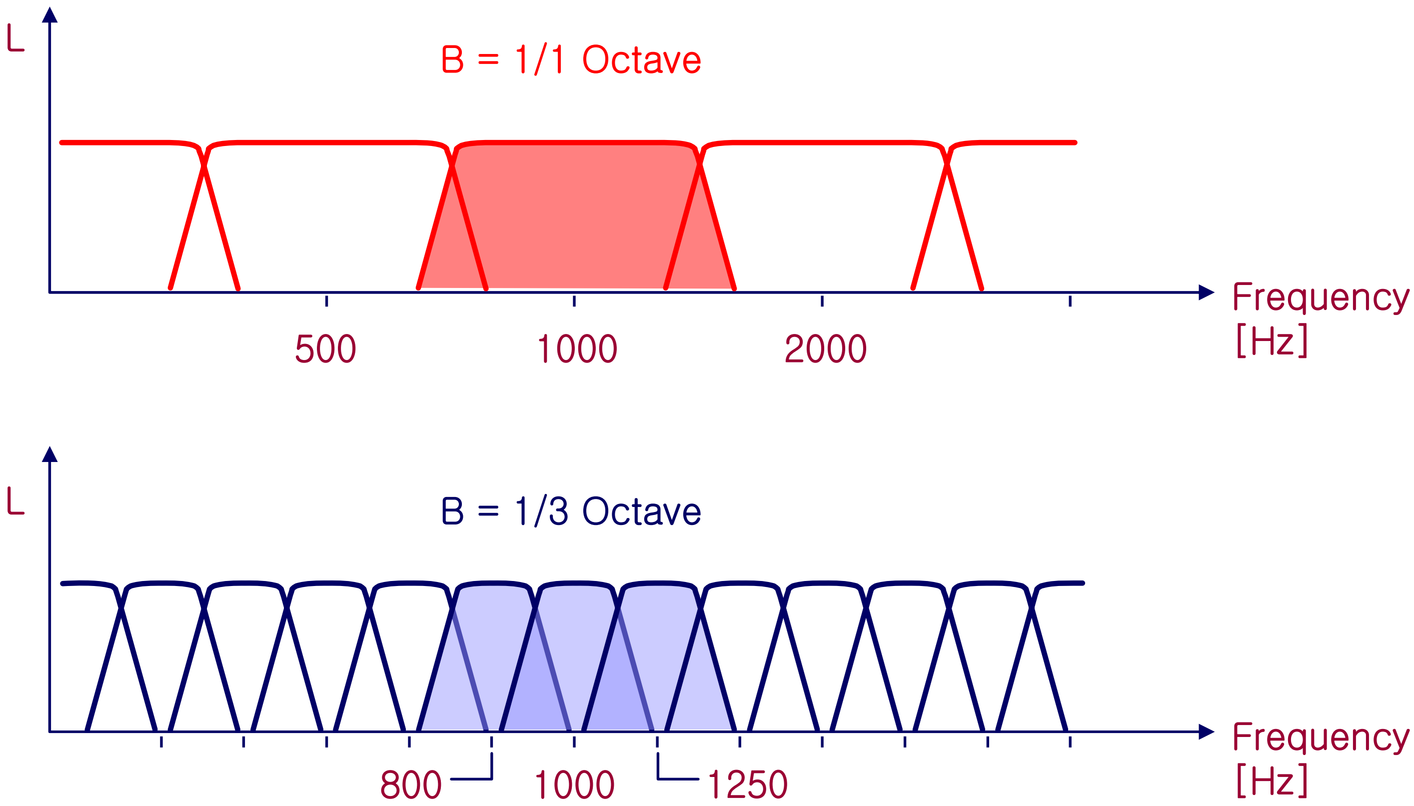


1/3 Octave

$$f_2 = \sqrt[3]{2} \times f_1 = 1.25 \times f_1$$

$$B = 0.23 \times f_0 \approx 23\%$$

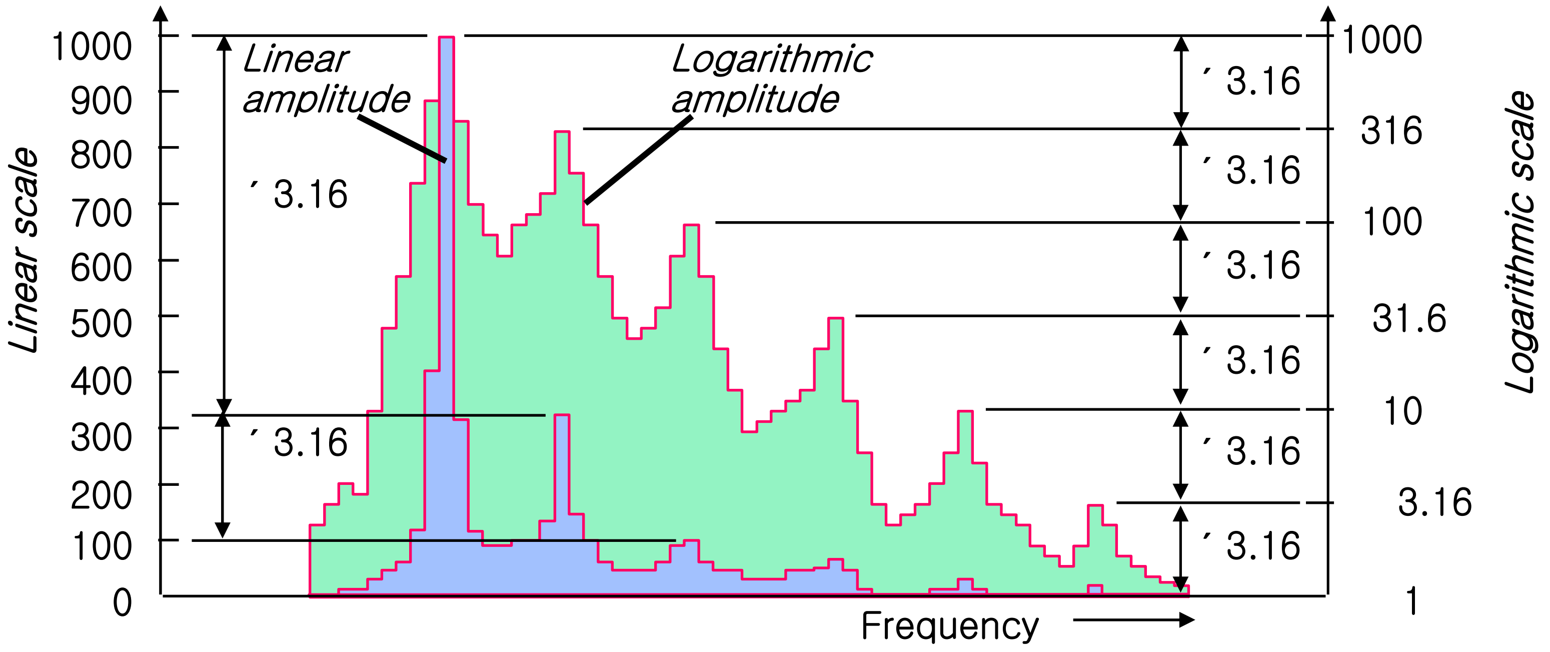
- 3 × 1/3 Oct. = 1/1 Oct.



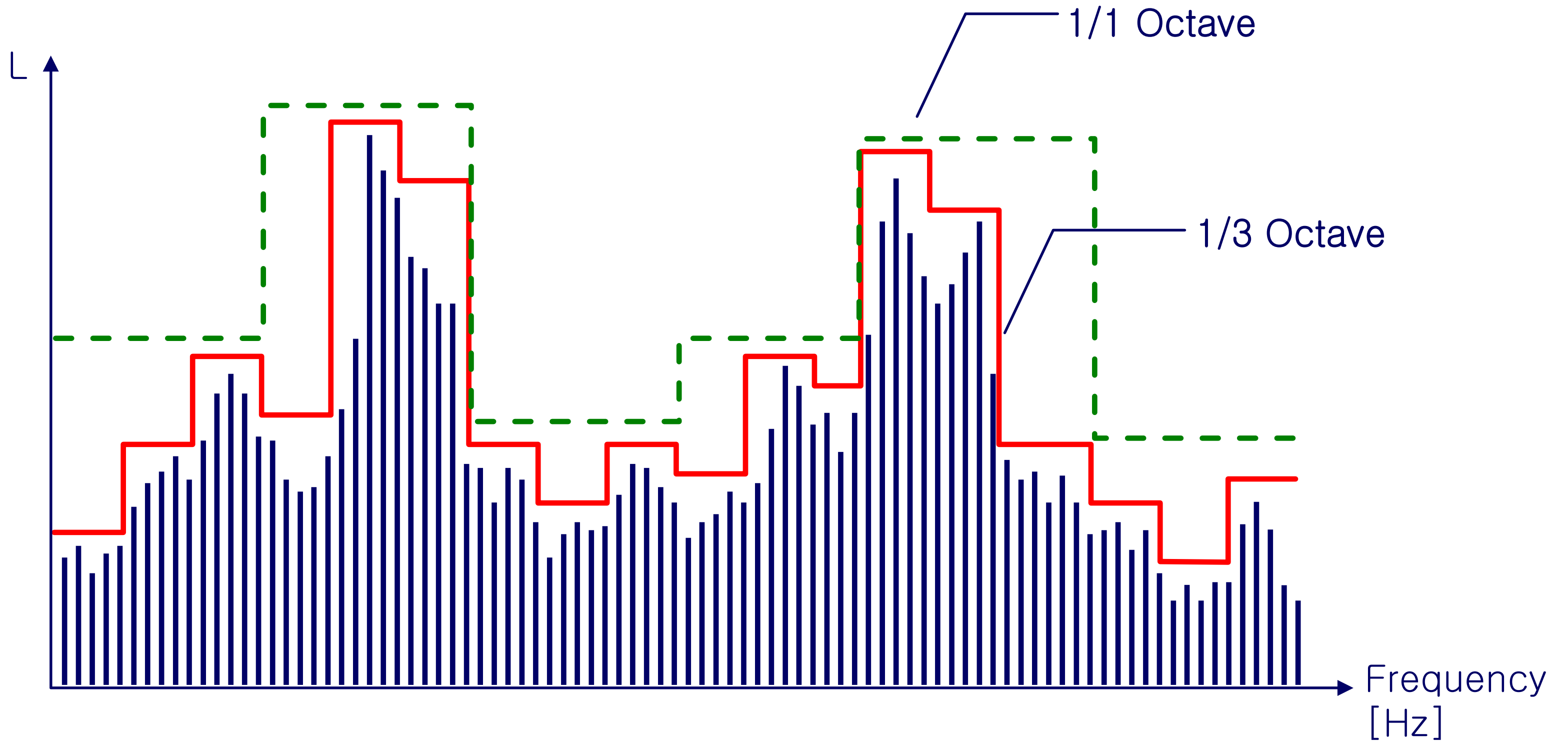
- Linear vs Logarithmic Amplitude Scales

Advantages of logarithmic amplitude scale

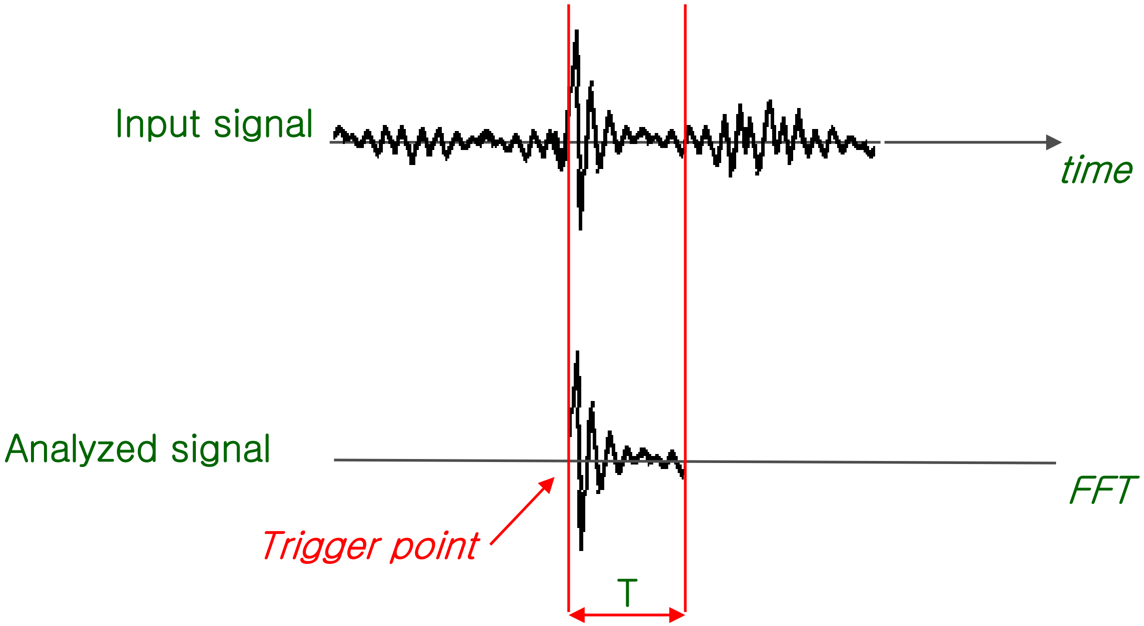
- Constant factor changes are equally displayed for all levels
- Optimal way of displaying a large dynamic range



- The Spectrogram

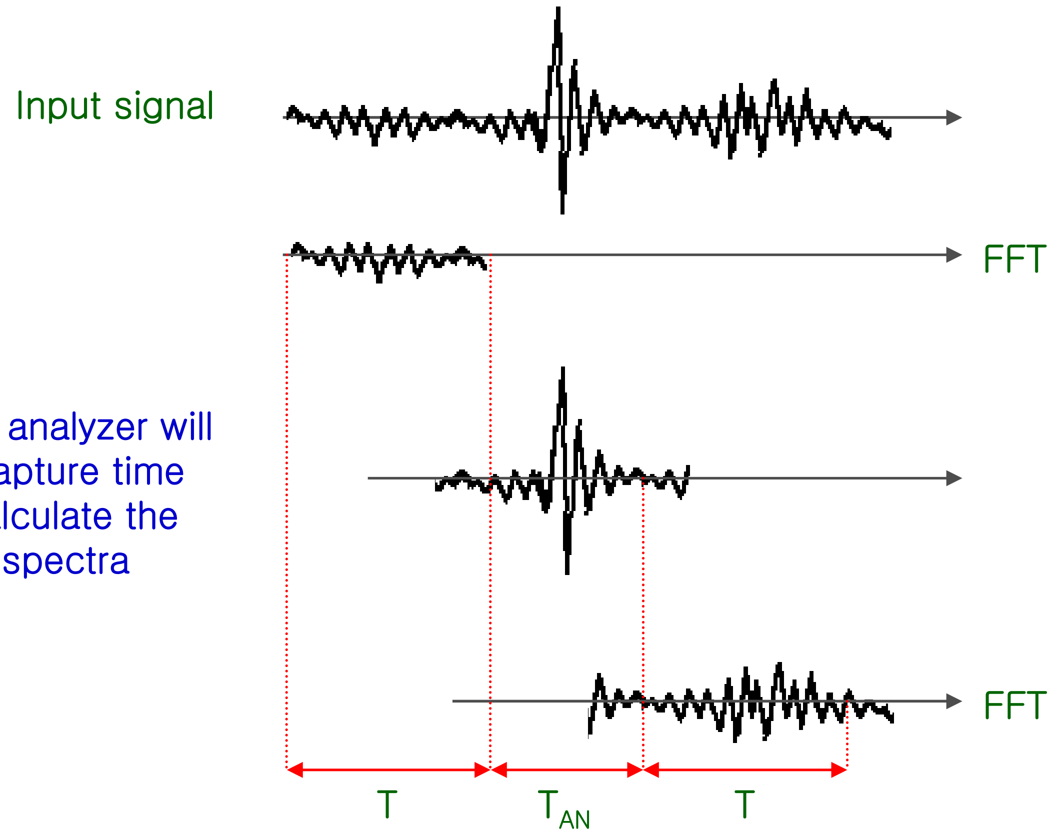


- Triggering



To select the part of the signal to be analyzed

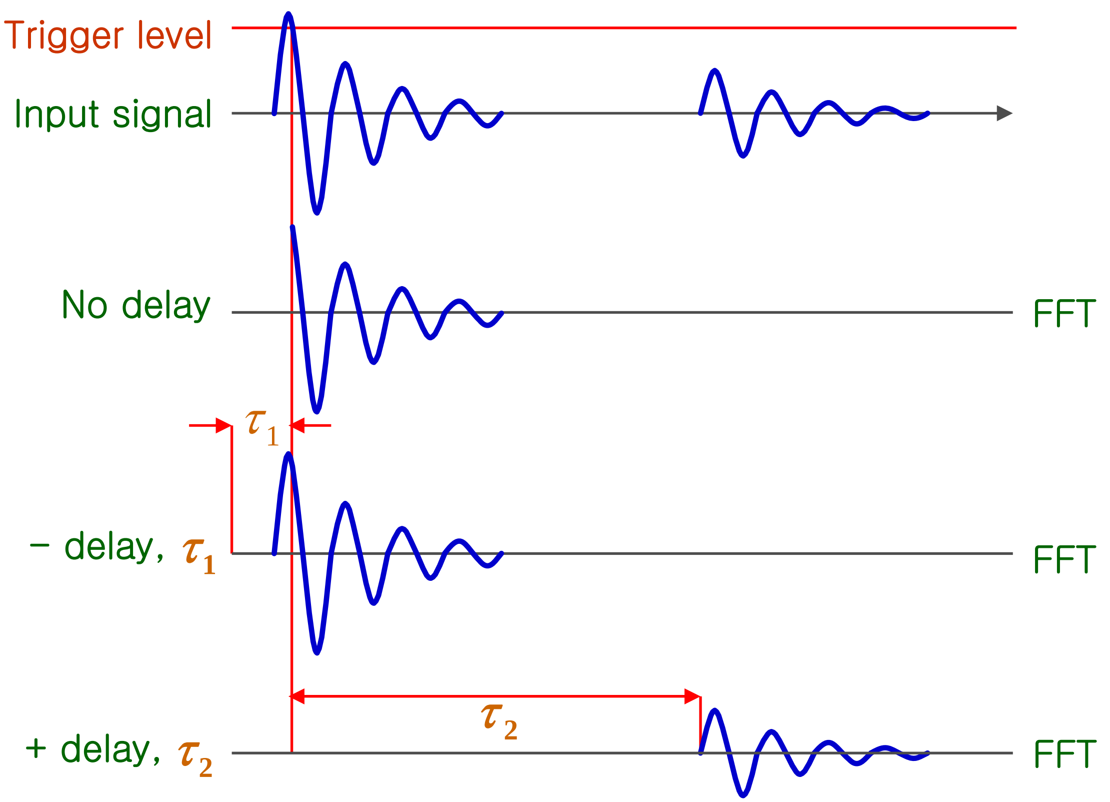
- Free Run (No Trigger)



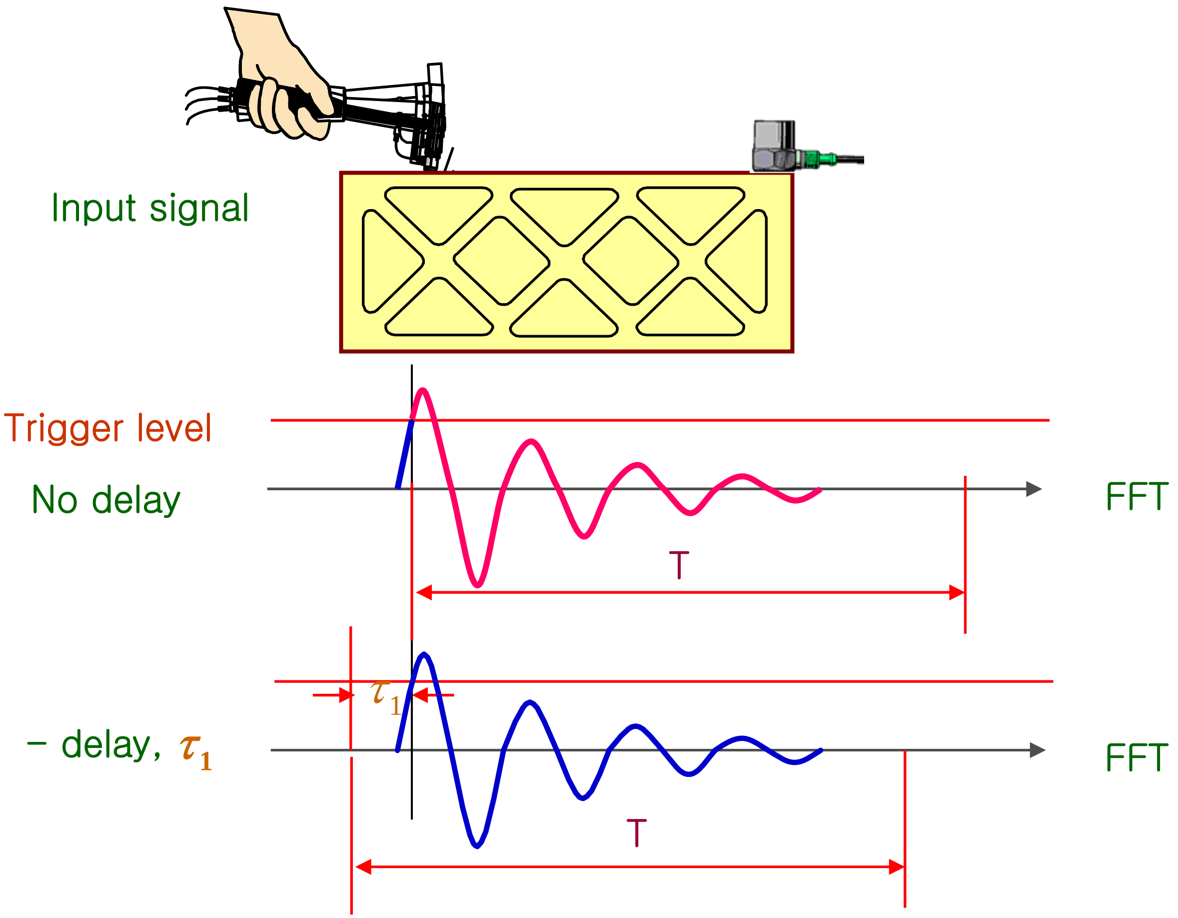
In free run, the analyzer will continuously capture time records and calculate the corresponding spectra

T_{AN} : analysis time

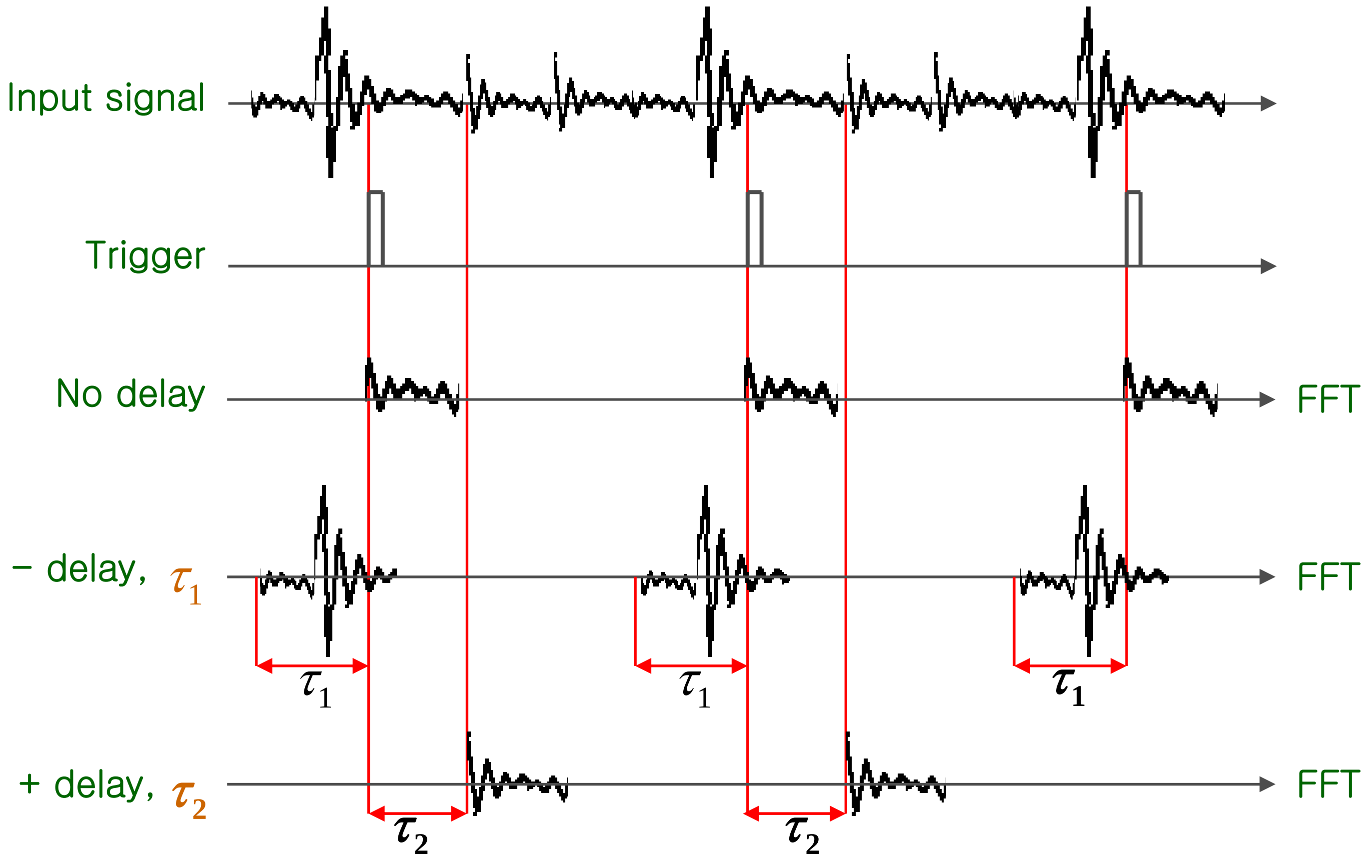
- Internal Trigger (Channel Trigger)



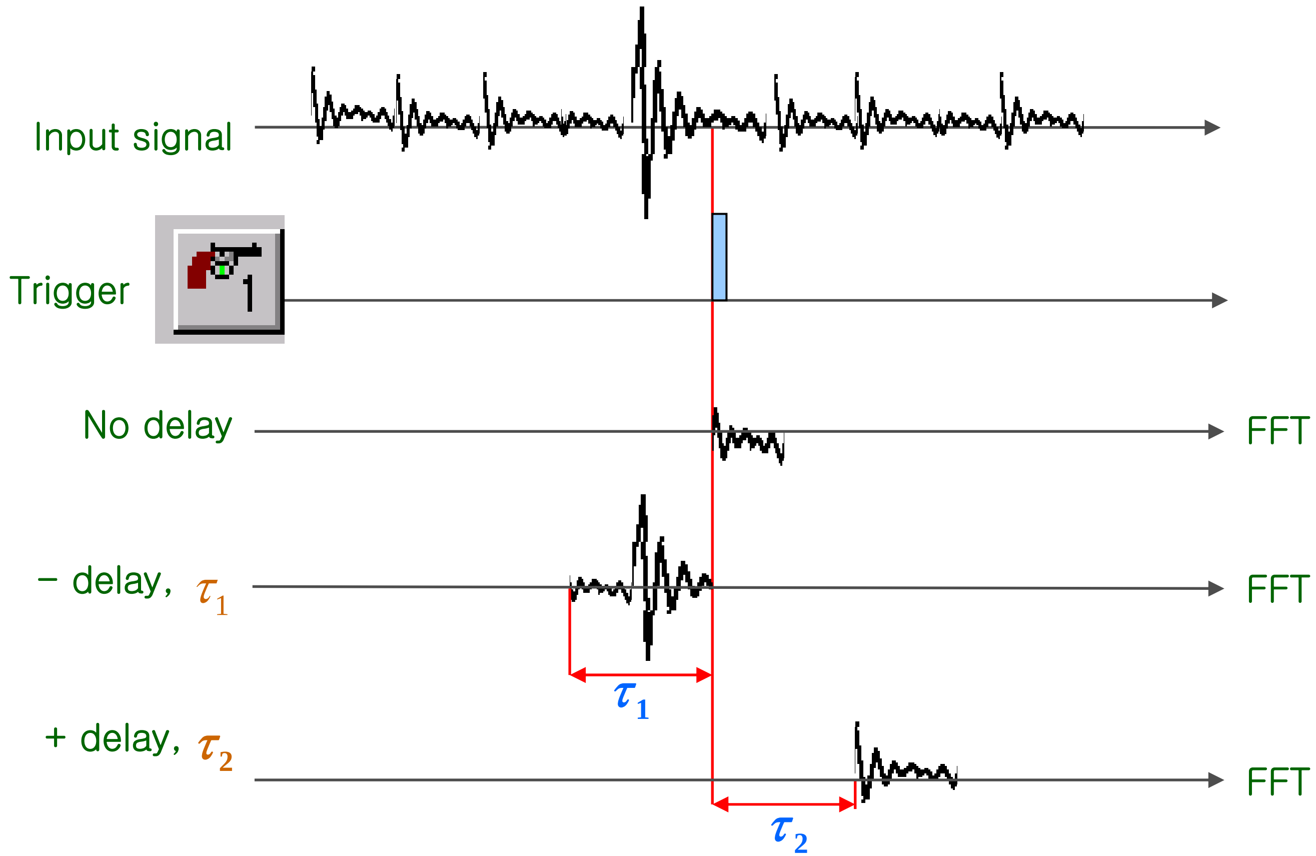
- Trigger Delay Example



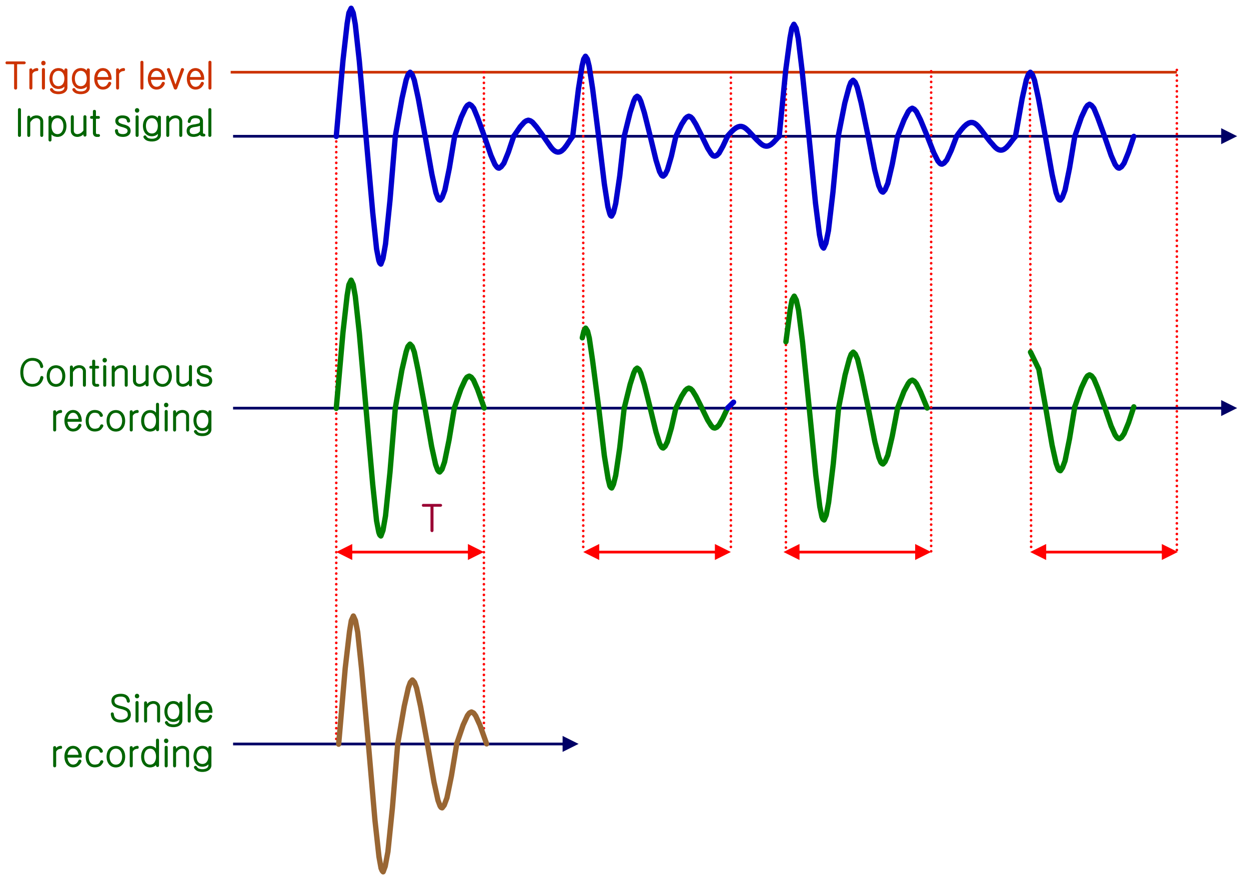
- External Trigger



- Manual Trigger



- Continuous / Single Recording



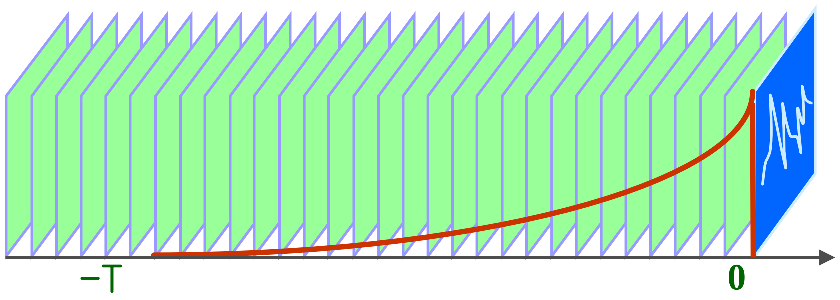
ex) shock and vibration measurement in transit

- Linear and Exponential Average

Lin. average in FFT analysis

Avg. No	1	2	3	…	N
Time	Record 1	Record 2	Record 3	…	Record N
Fourier spec.	Spectrum 1	Spectrum 2	Spectrum 3	…	Spectrum N
Auto spec.	S_1	$(S_1+S_2)/2$	$(S_1+S_2+S_3)/3$	…	Sum S_N/N

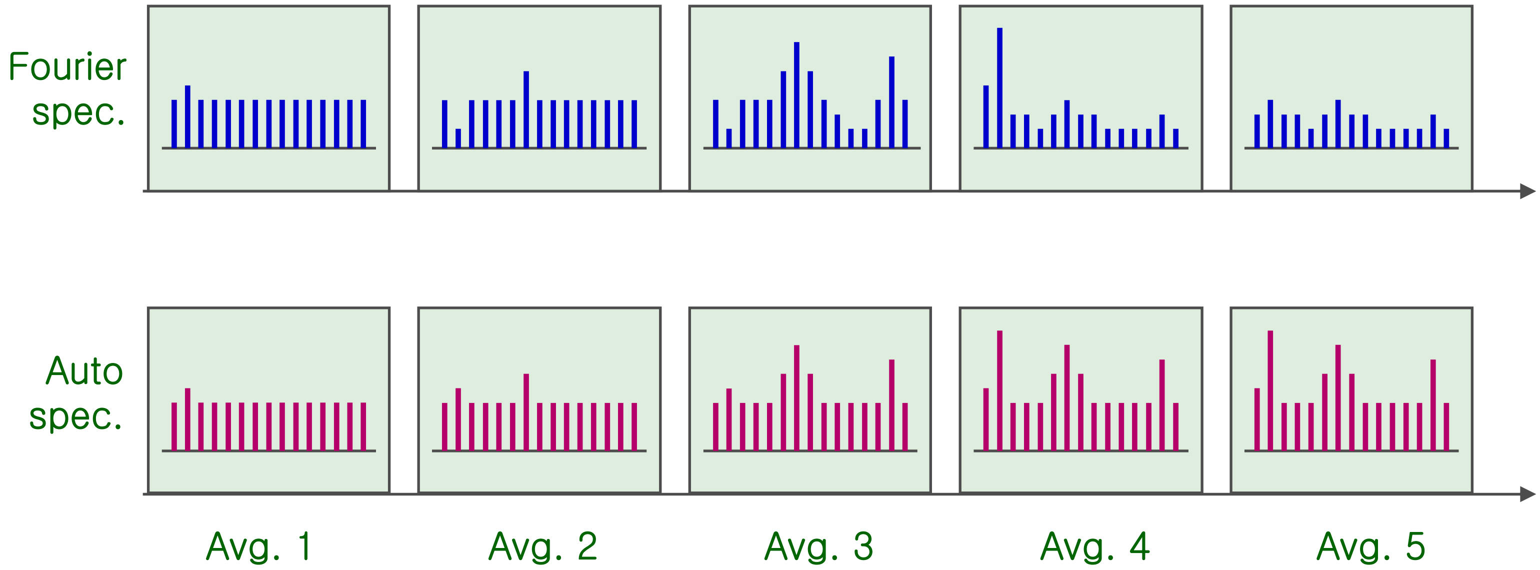
Exp. average in FFT analysis



$$e^{-t}$$

- Peak Hold Average(I)

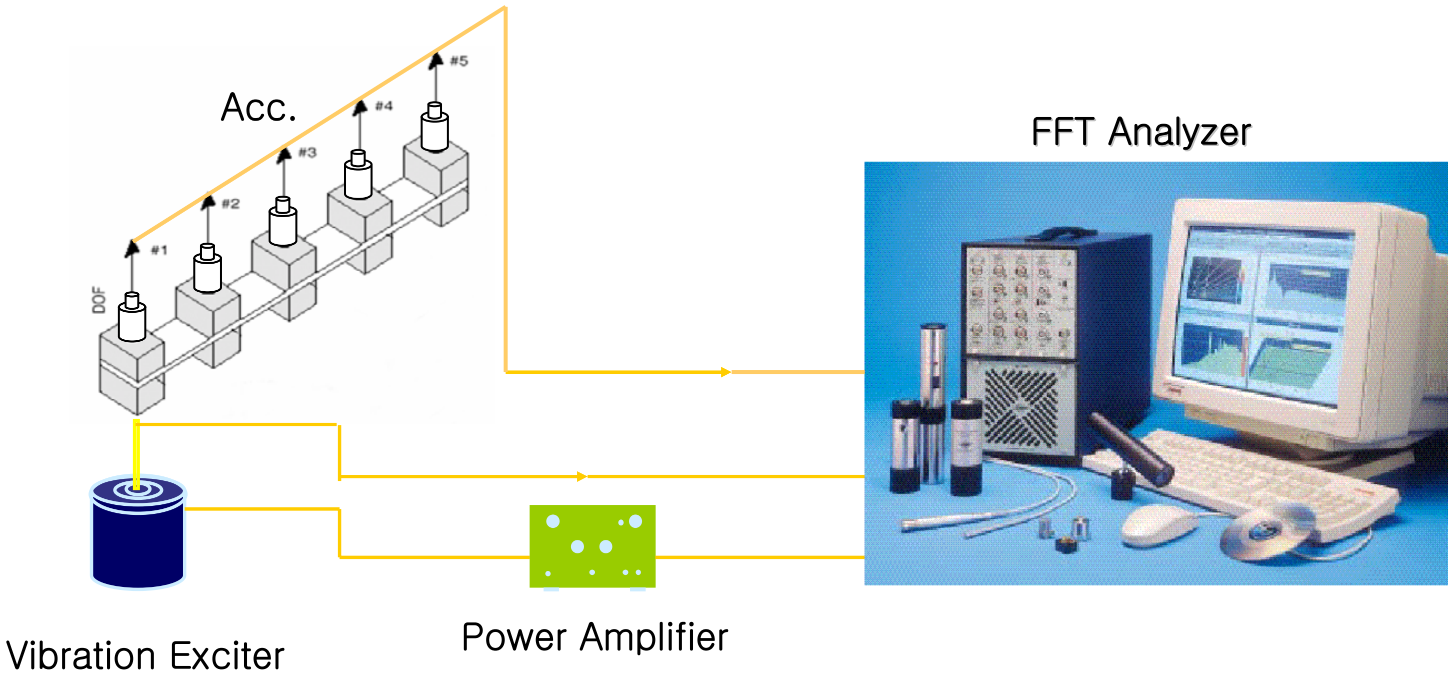
Peak average in FFT analysis



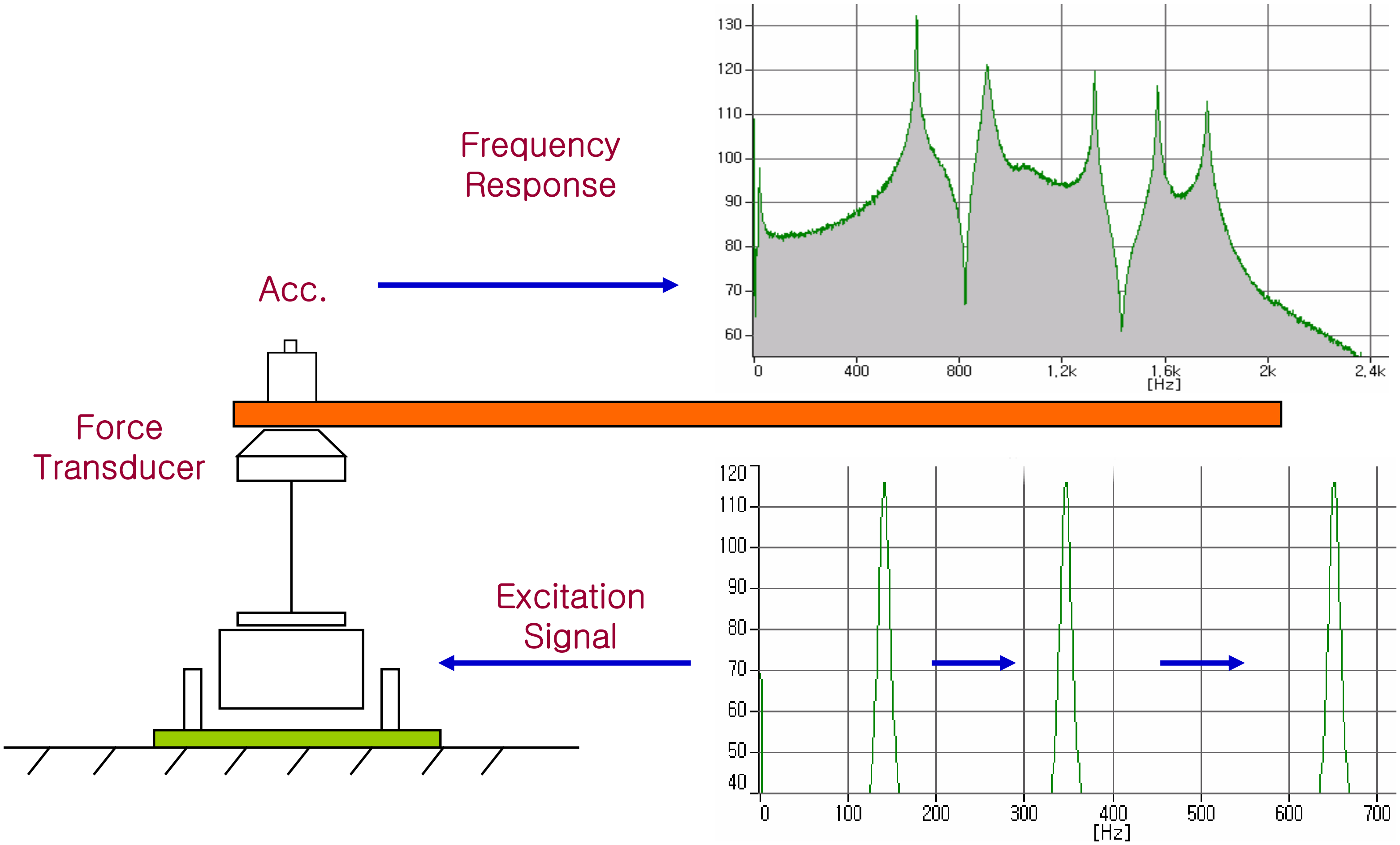
- Peak Hold Average(II)

Shaker excitation – Increase Power

– Measure Non-Linear Properties

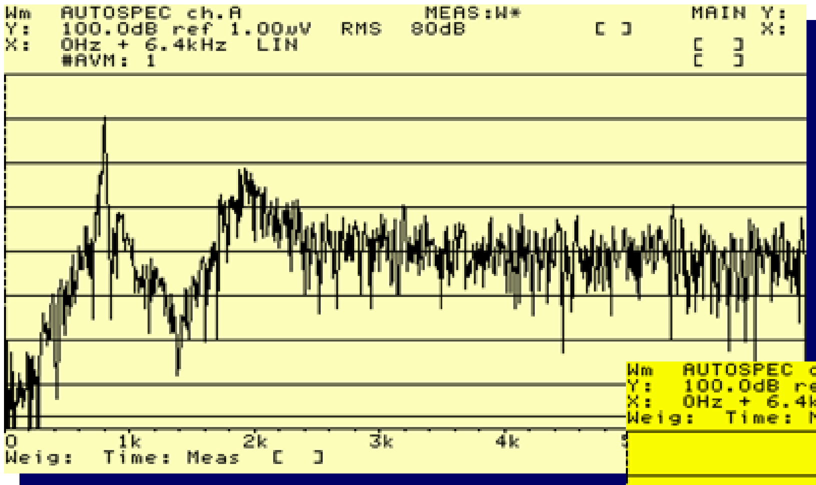


- Peak Hold Average(III)



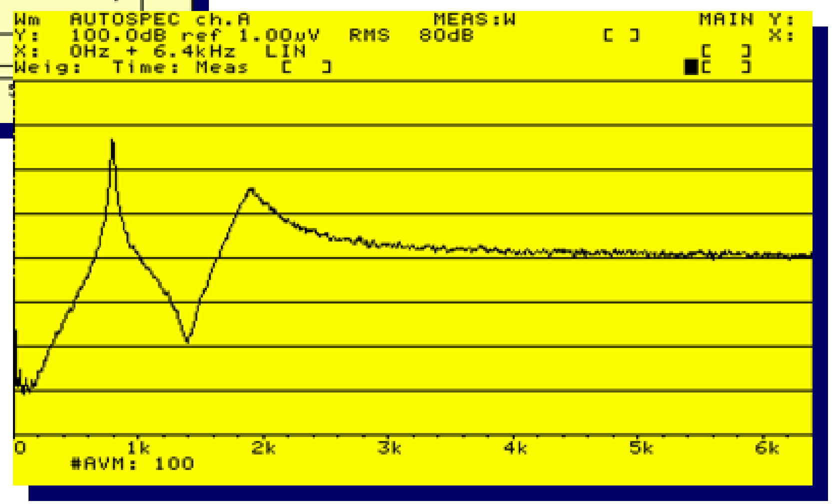
- Swept Excitation with a constant amplitude ⇒ response characteristic

- Reason for averaging signal



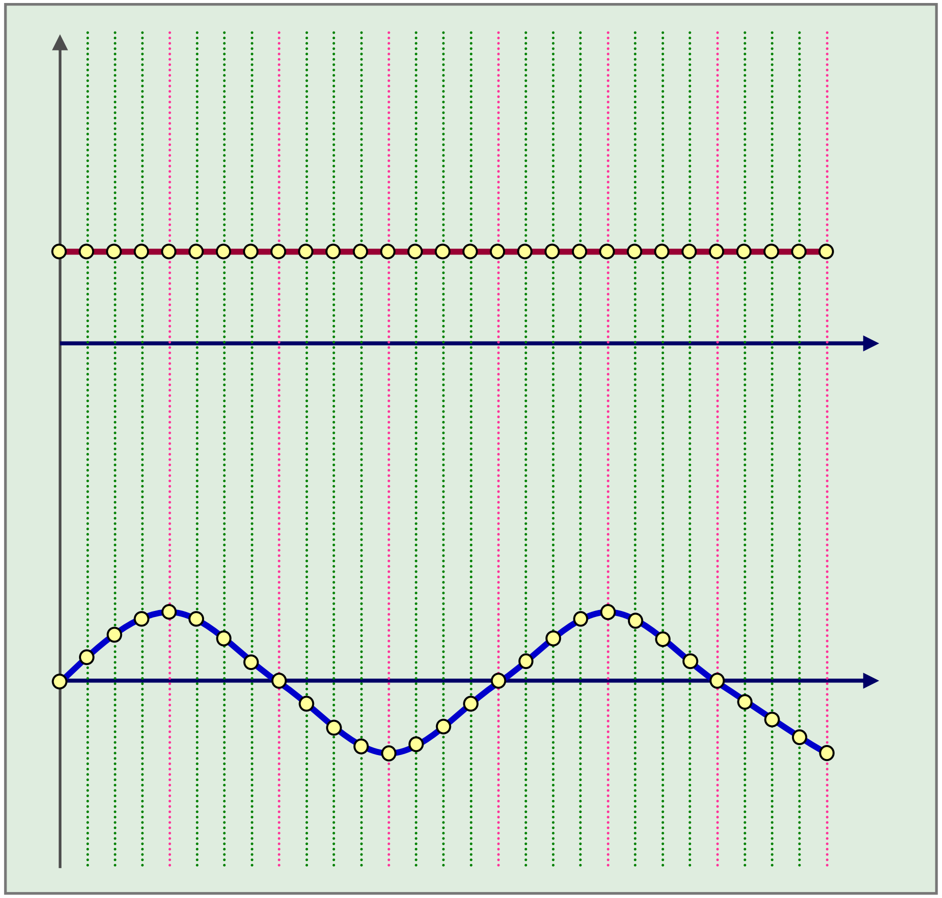
No average

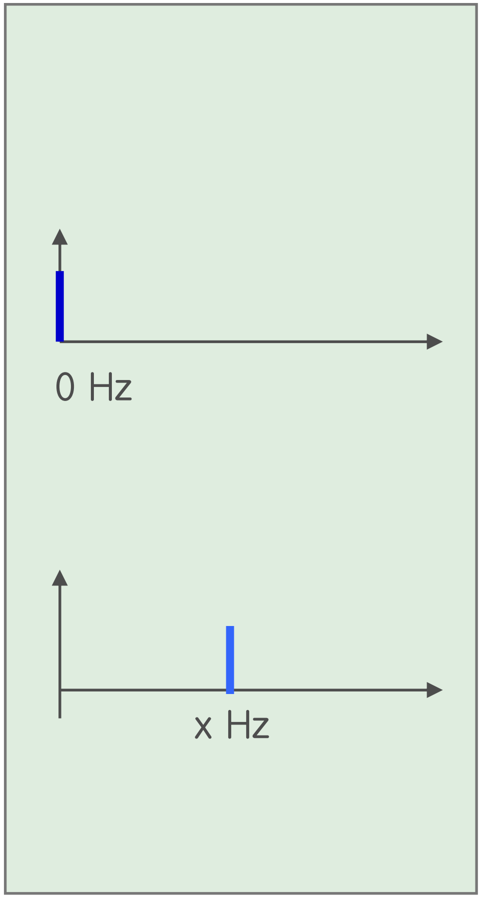
100 Spectrum average



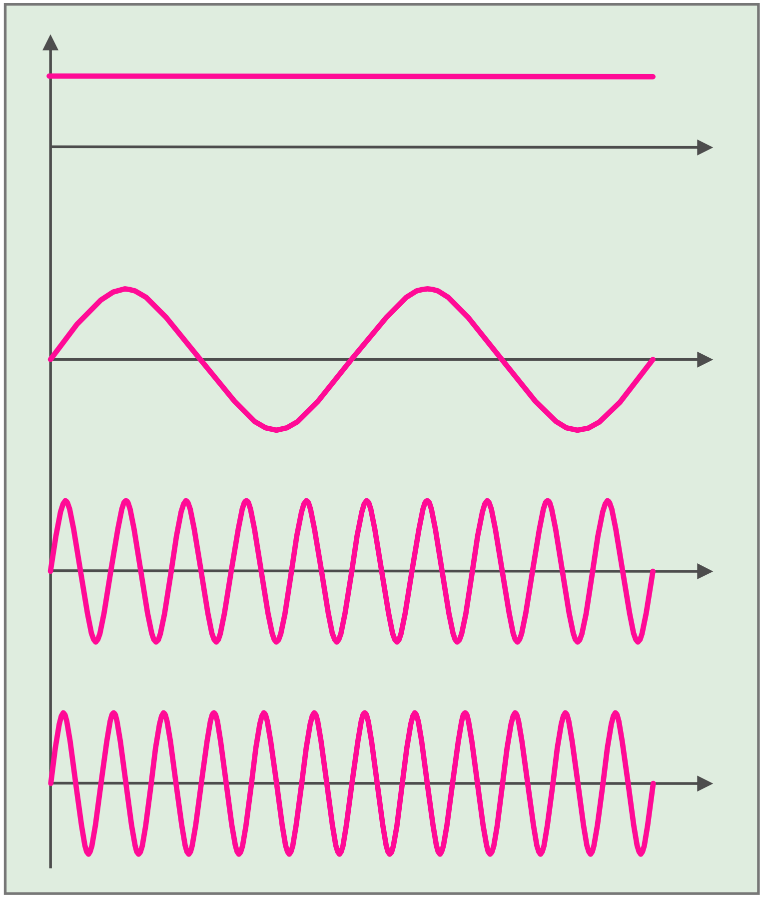
- Averaging signal
  - Reason for averaging signal
    - reduce random error
  - Time domain average
    - Fast, Slow, Impulse (Sound level meter)
  - Frequency domain average
    - Lin, Exp, Peak

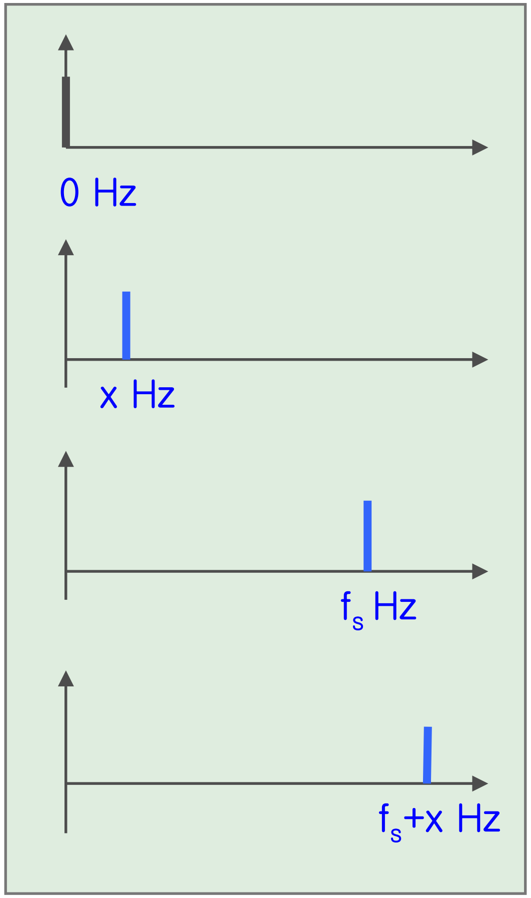
- Sampling (I)



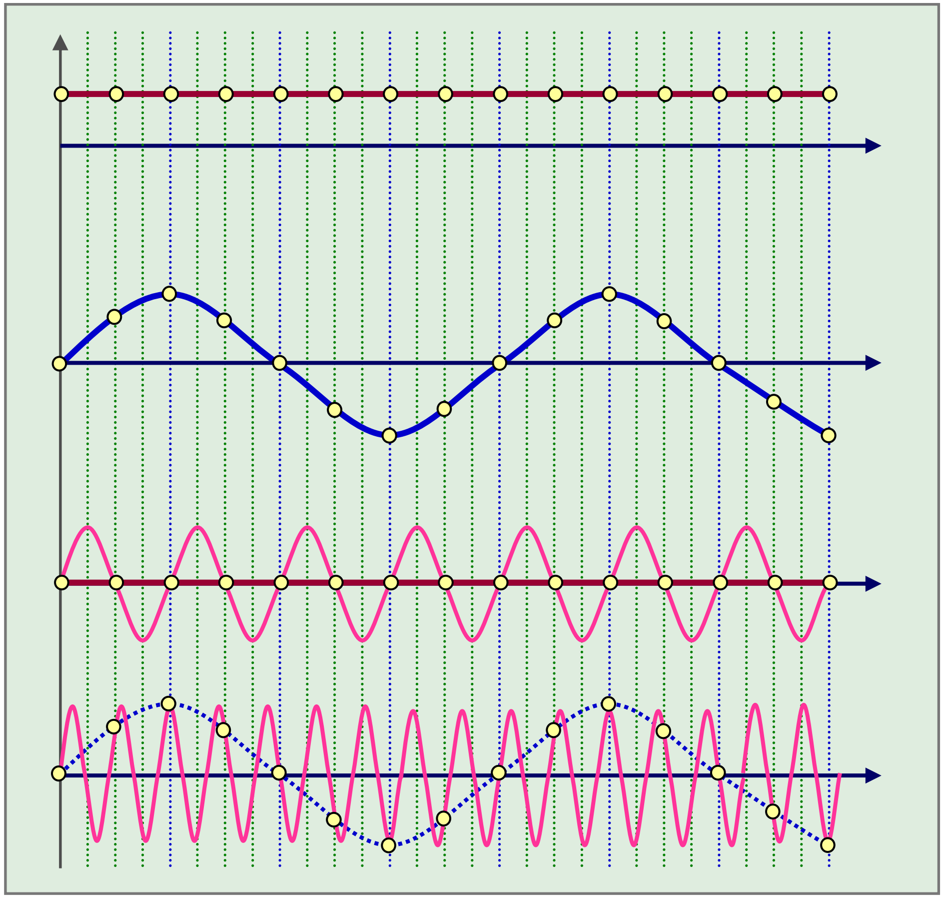


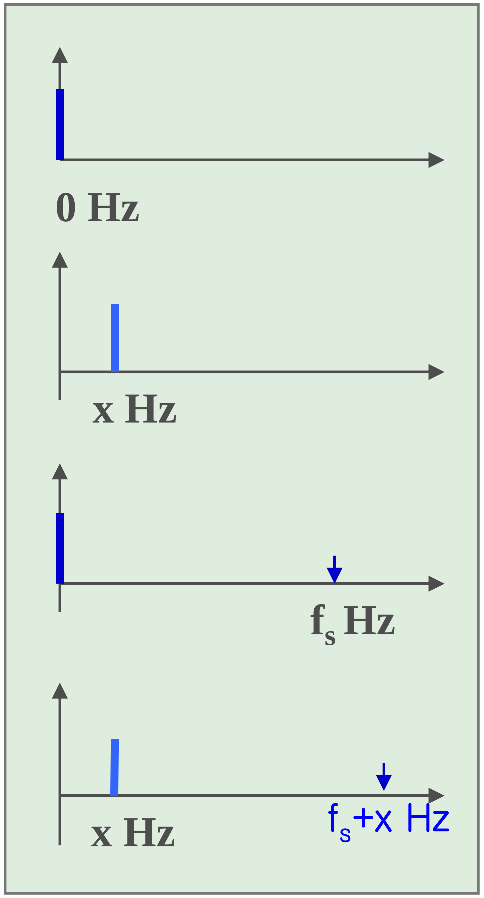
- Sampling (II)



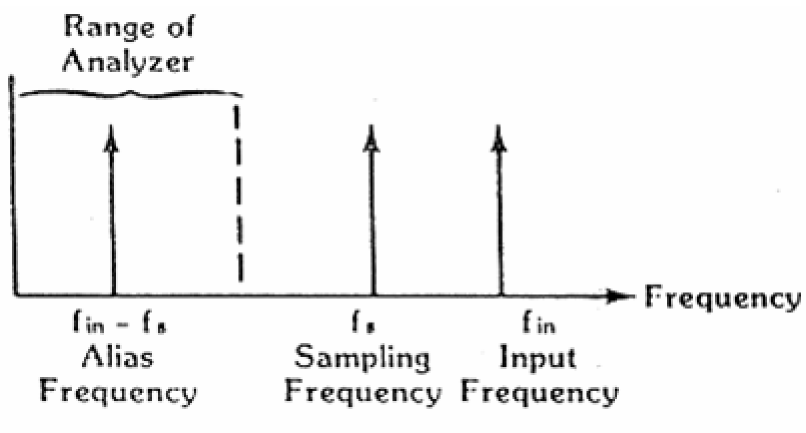


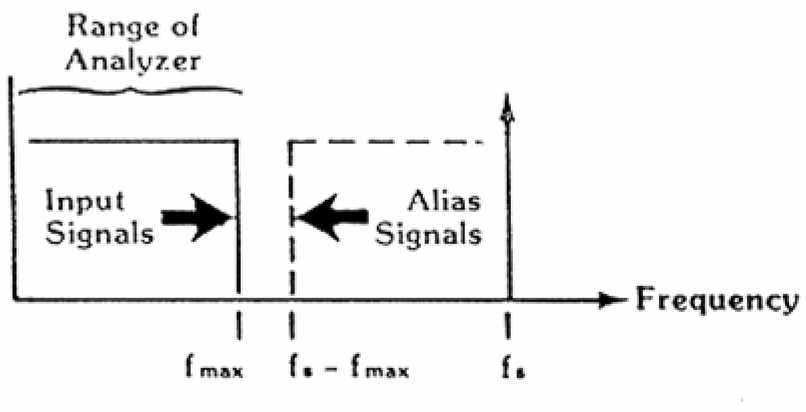
- Aliasing (Sampling)



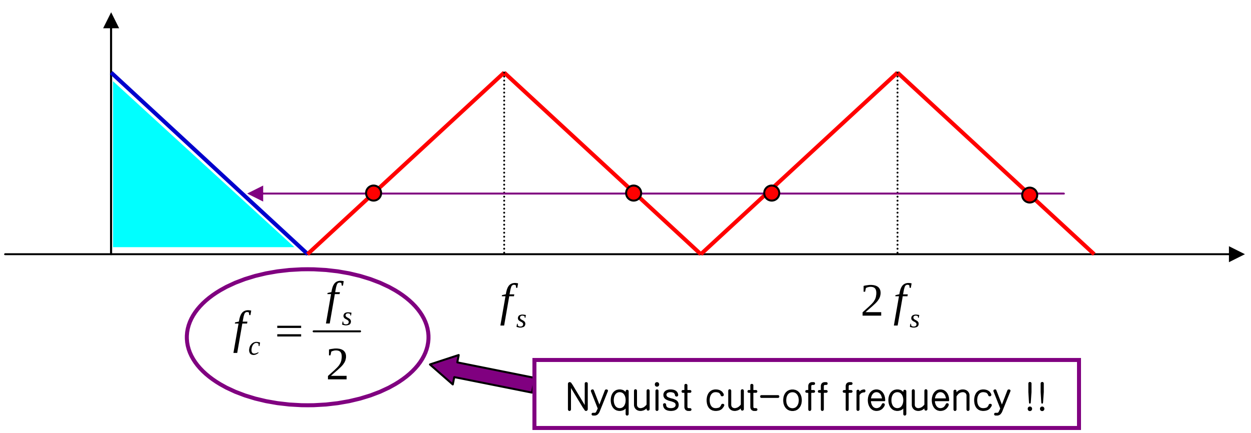


- Aliasing Frequyency

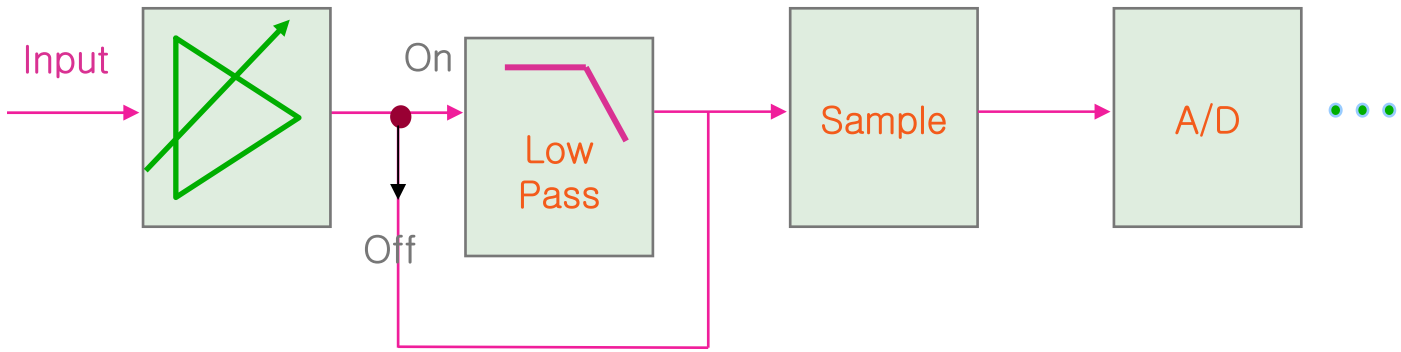


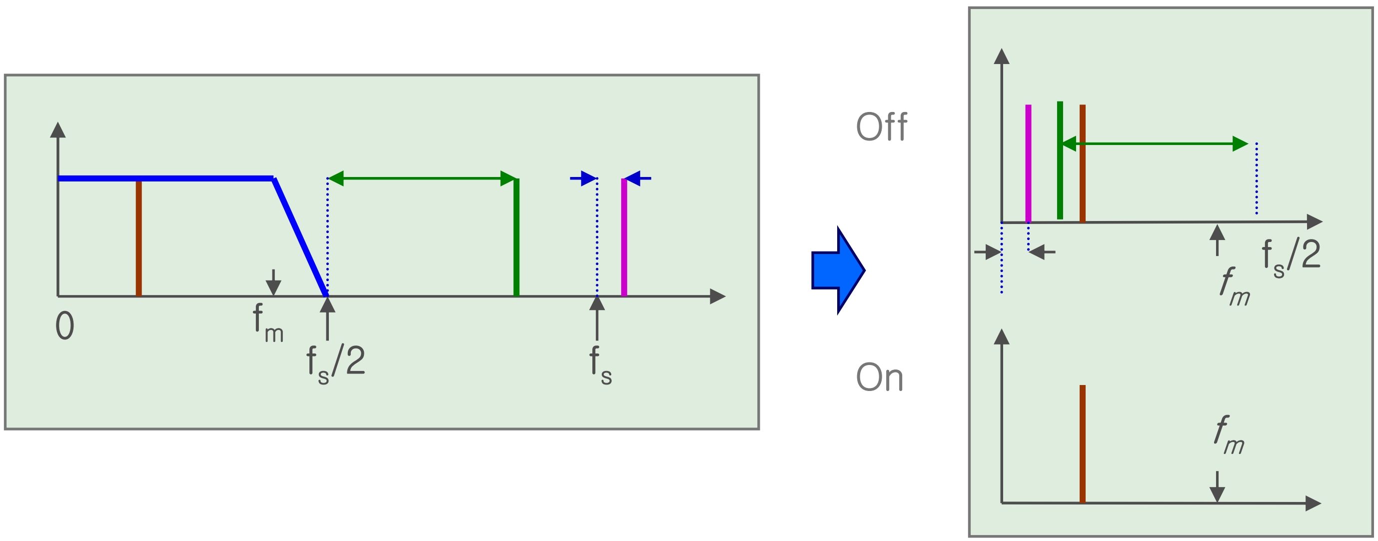


- Accordion-pleated fashion



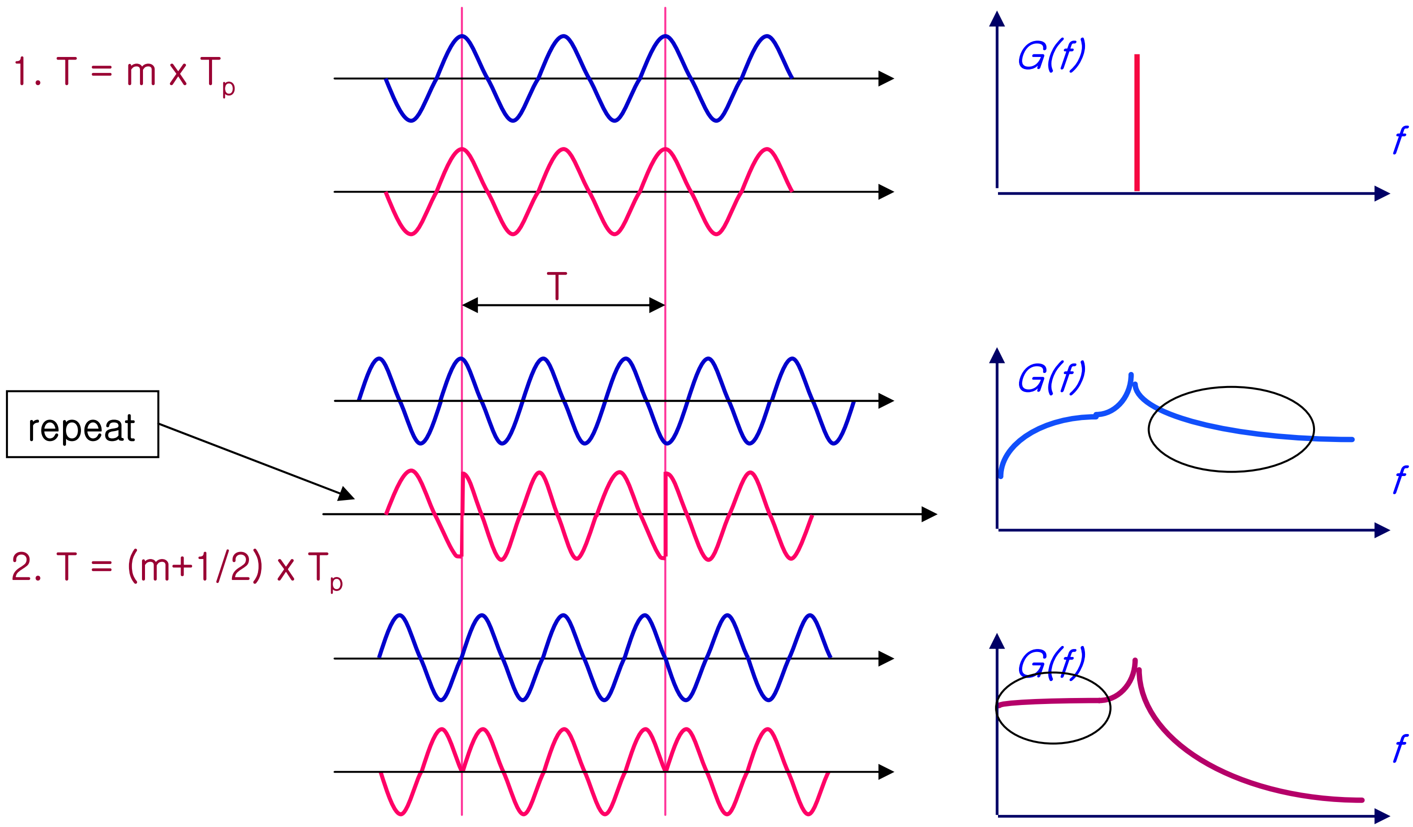
- Anti-aliasing Filter





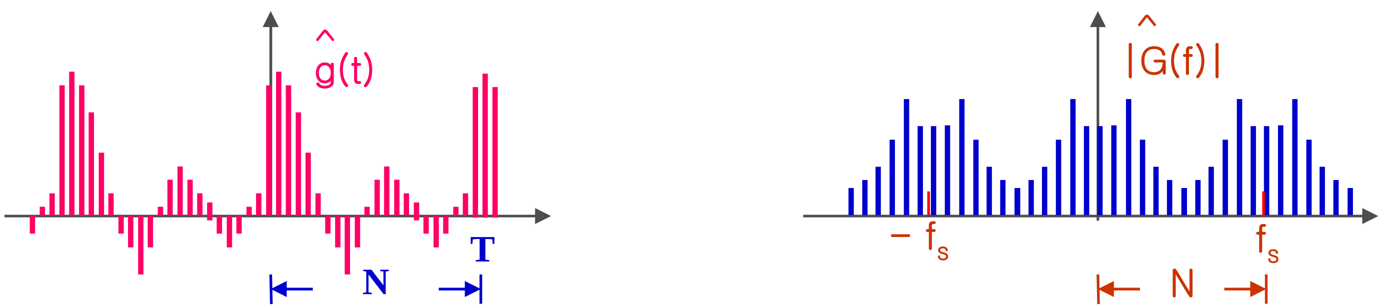
- Leakage

$G(t) = \cos 2\pi t/T_p$



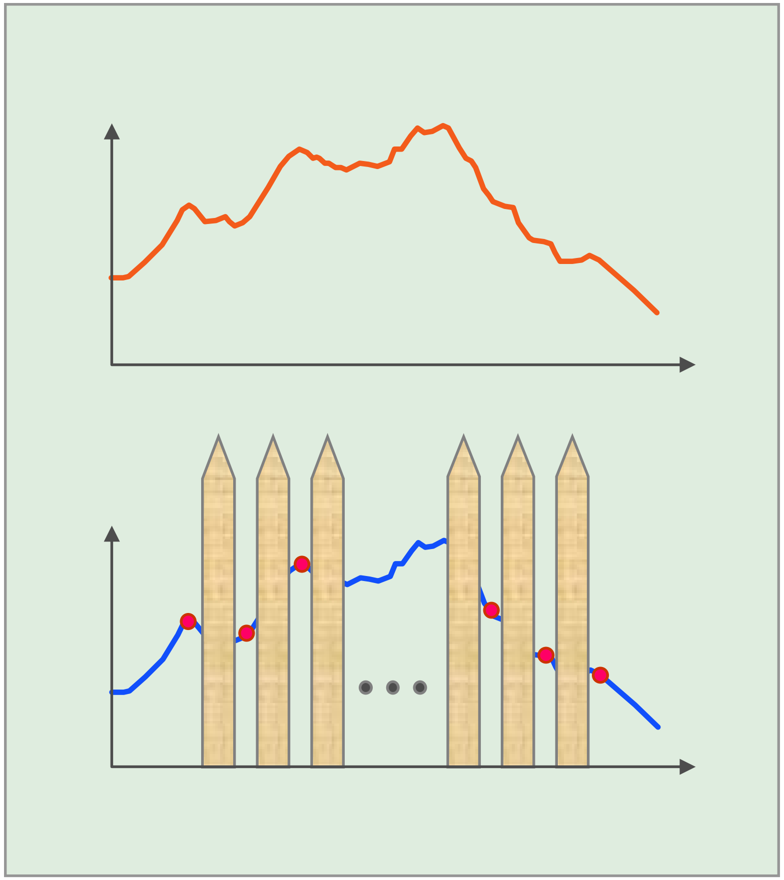
- Sampling in Frequency Domain

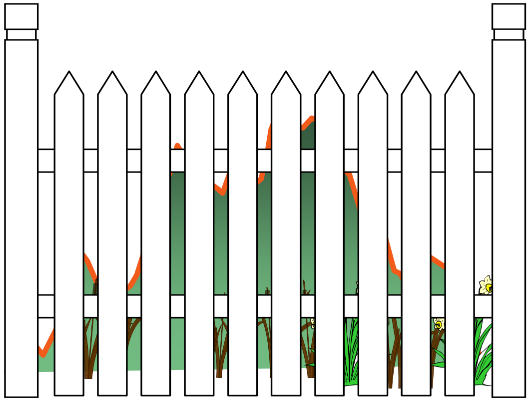
$$\Delta f = 1 / T$$
$$f_s = N \times \Delta f$$



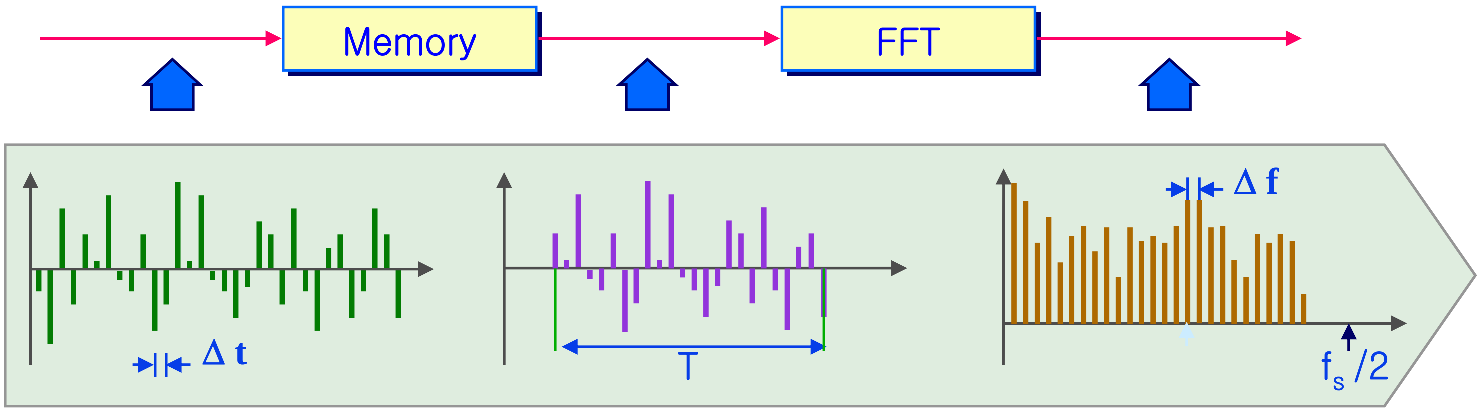
•The fact that the spectrum has become discrete is called the picket fence effect.

- Picket Fence Effect





- Parameters



$$T = N \times \Delta t$$

$$\Delta t = 1/f_s$$

$$f_s = N \times \Delta f$$

$$\Rightarrow \Delta f \times T = 1$$

T = Record time

N = Number of samples

Δt = Sampling interval

f_s = Sampling frequency

Δf = Frequency resolution

• Picket Fence Effect – Example

Ex1. 1600 Lines

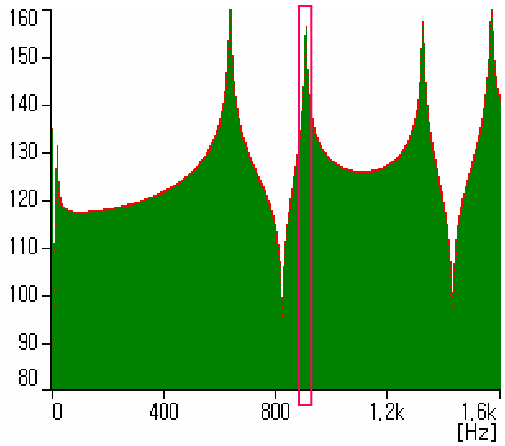
1600 Hz

1Hz ⇒ Δf

Ex2. 400 Lines

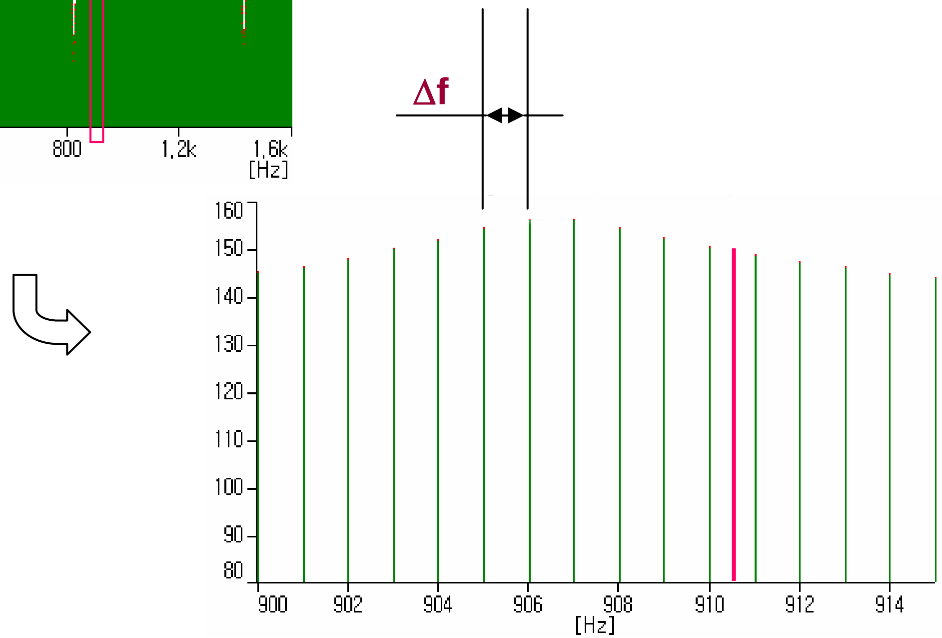
800 Hz

2Hz ⇒ Δf

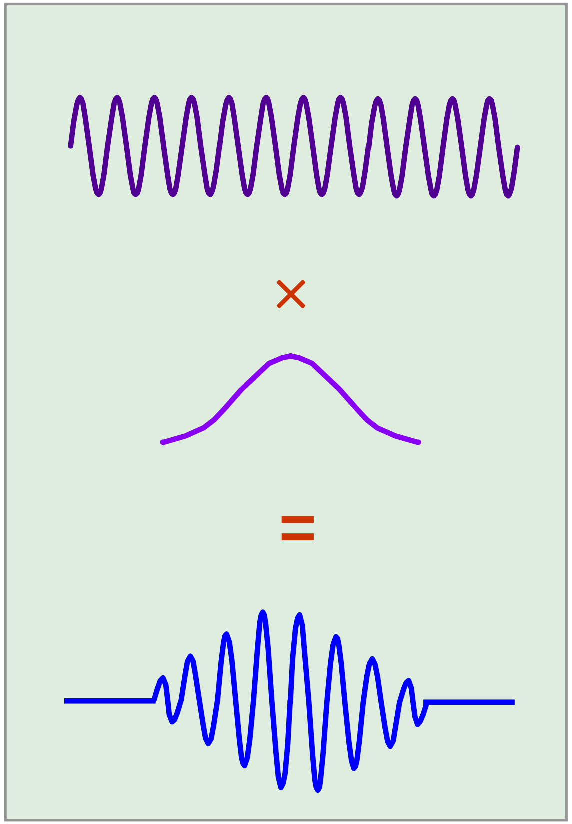


If I want to see 910.5 Hz Component.

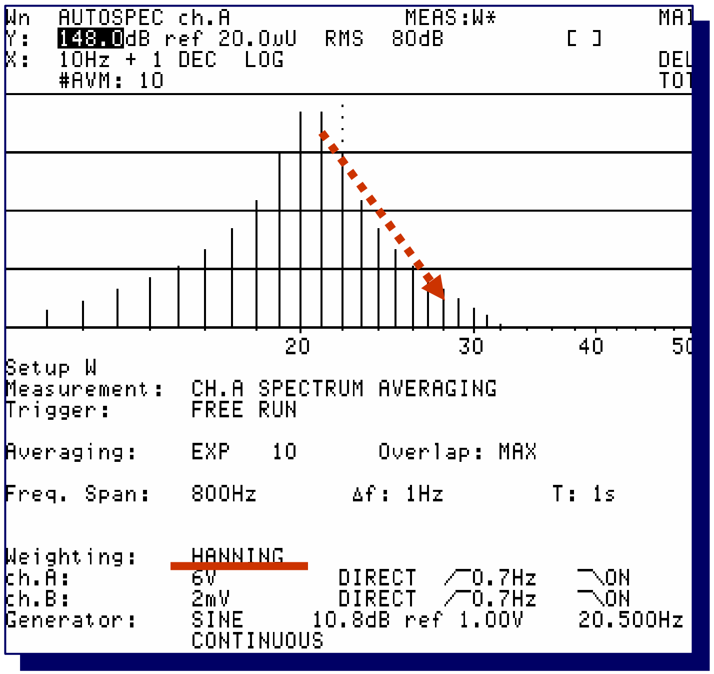
Possible or Impossible ?

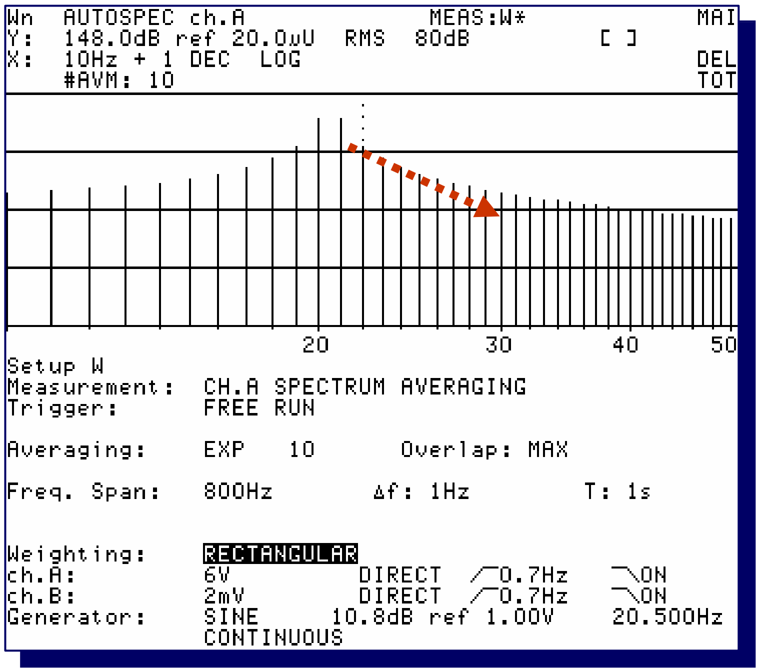


- Weighting function
  - Rectangular (No weight)
  - Hanning
  - Kaiser-Bessel
  - Flat Top
  - Transient
  - Exponential
  - User Defined

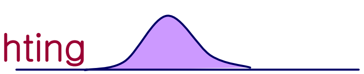


- Comparison of the spectra of the Flat(Rectangular) and Hanning time weighting functions





Hanning Weighting



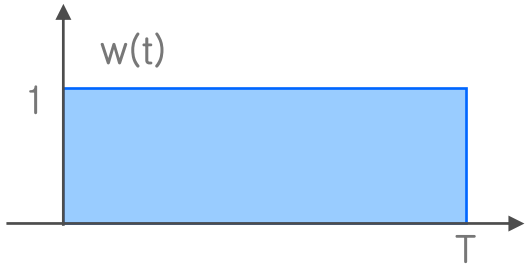
Rectangular Weighting

Recorder length = 1 sec
Input signal= 20.5 Hz Sine

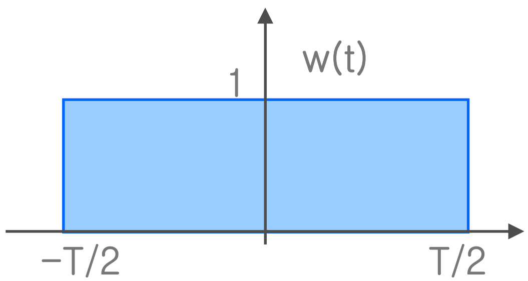
- Weighting function – Rectangular Window

To call Uniform window, No Window, or Flat window.

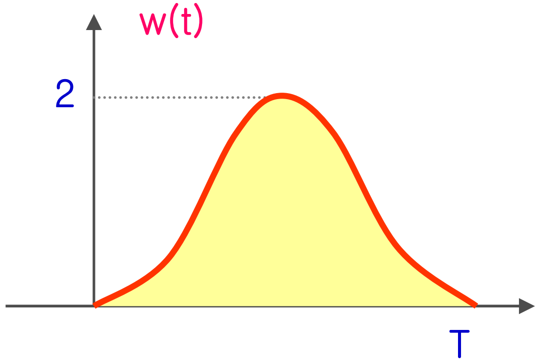
$$w(t) = 1; 0 \le t < T$$



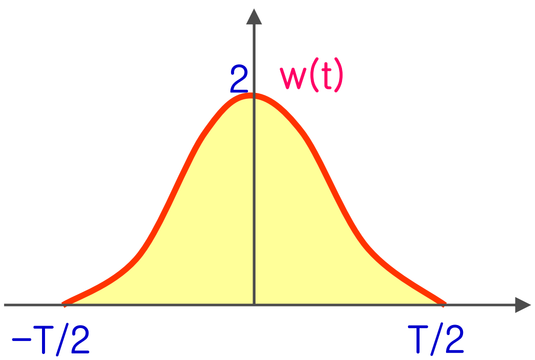
$$w(t) = 1, -T/2 \le t < T/2$$



- Weighting function – Hanning Window



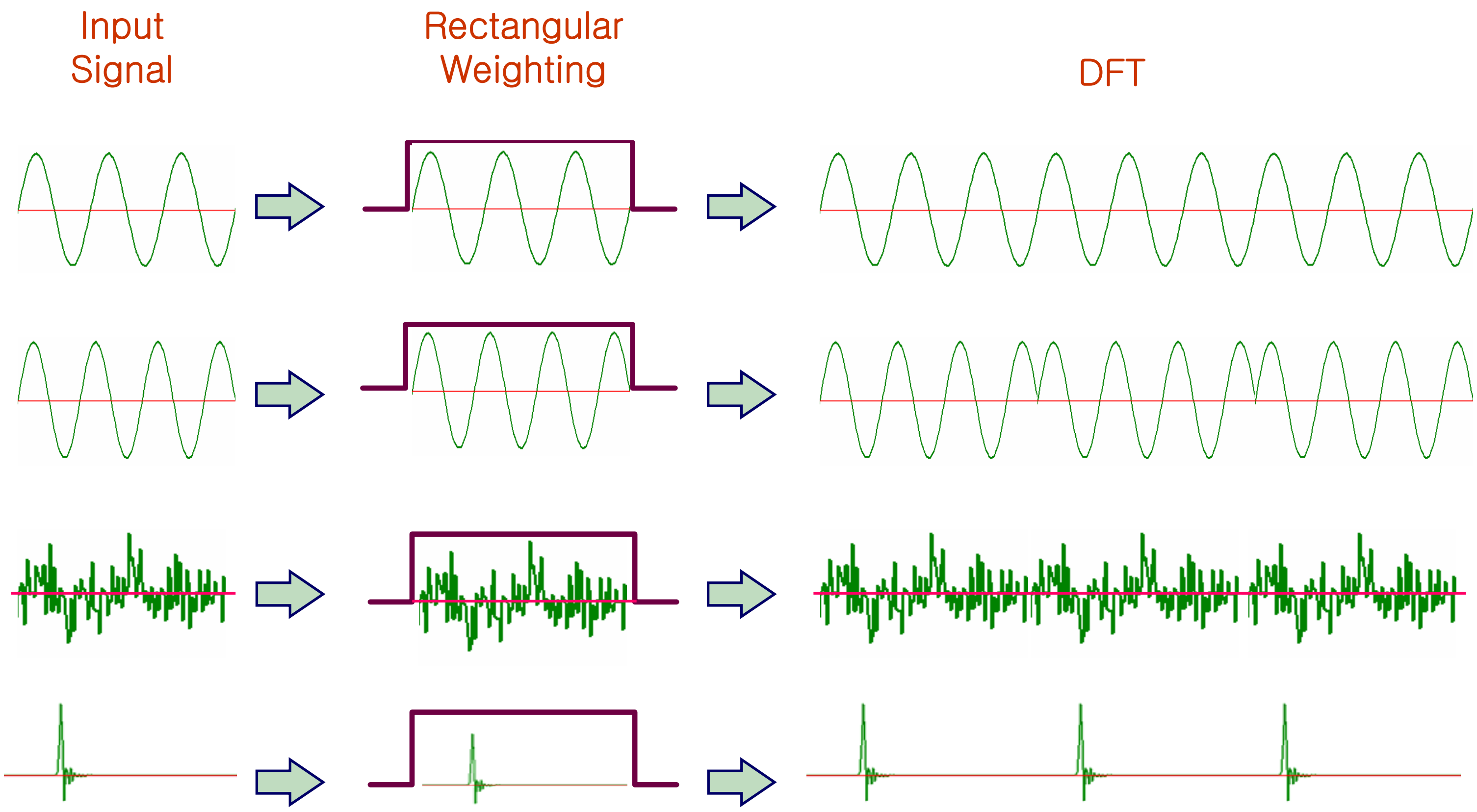
$$w(t) = 1 - \cos\frac{2\pi t}{T}; 0 \le t < T$$
$$= 2\sin^2\frac{\pi t}{T}; 0 \le t < T$$



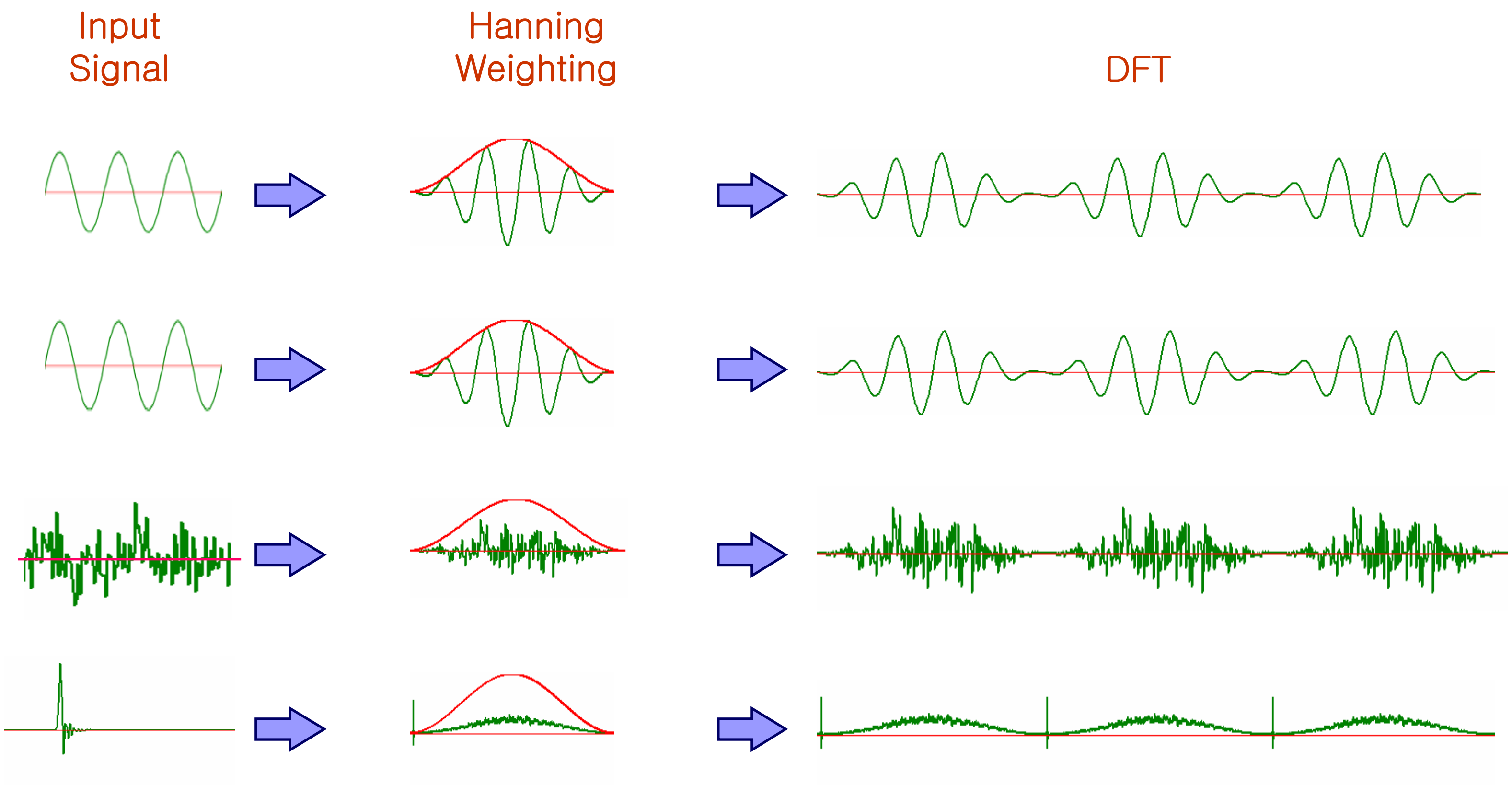
$$w(t) = 1 + \cos\frac{2\pi t}{T}; -T/2 \le t < T/2$$
$$= 2\cos^2\frac{\pi t}{T}; -T/2 \le t < T/2$$

To reduce the effect the leakage, one of the most common is the Hanning window which consist of a period of a cosine lifted up to have a minimum value of 0.

- Weighting function – Rectangular Weighting in DFT

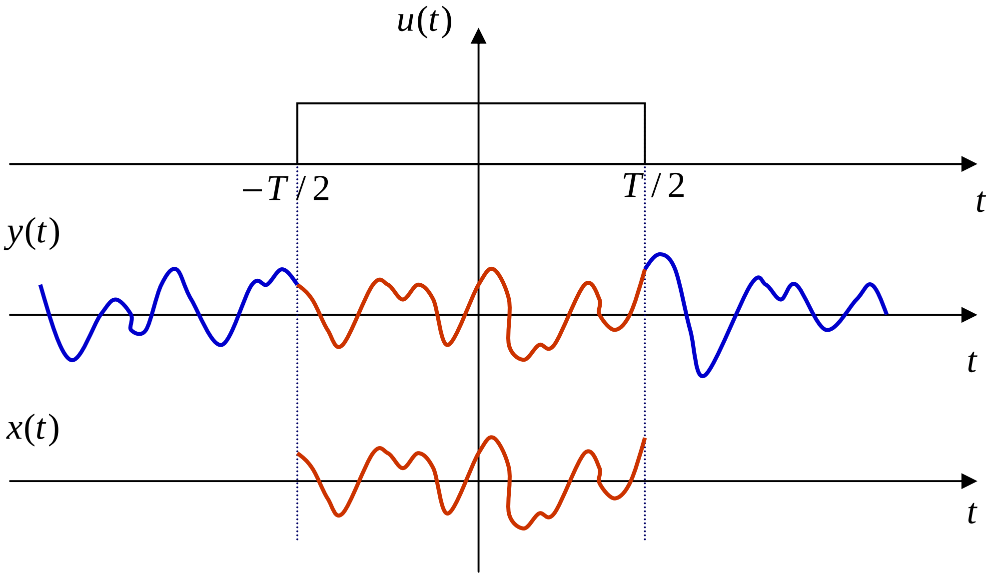


- Weighting function – Hanning Weighting in DFT



The “Hanning” time weighting function will always modify the signal to be analyzed, but no abrupt transitions will be created at the ends of the time records.
The error will therefore be much smaller for stationary signals than it was, when the “flat” weighting was used. For transients the situation is just the opposite. The signal to be analyzed will now no longer contain all the energy and the result therefore be wrong.

- Rectangular Weighting –Boxcar function



- Estimate of Y(f)

$$X(f) = \int_{-T/2}^{T/2} x(t)\, e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} u(t)\; y(t)\, e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} Y(f-\xi) U(\xi) d\xi = U(f) * Y(f)$$

- Proof)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Y(f-\xi)U(\xi)d\xi\, e^{j2\pi ft}df = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Y(f-\xi)U(\xi)e^{j2\pi ft}d\xi\, df$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Y(f-\xi)e^{j2\pi(f-\xi)t}U(\xi)e^{j2\pi\xi t}d\xi\, df$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Y(f-\xi)e^{j2\pi(f-\xi)t}df\; U(\xi)e^{j2\pi\xi t}\; df$$

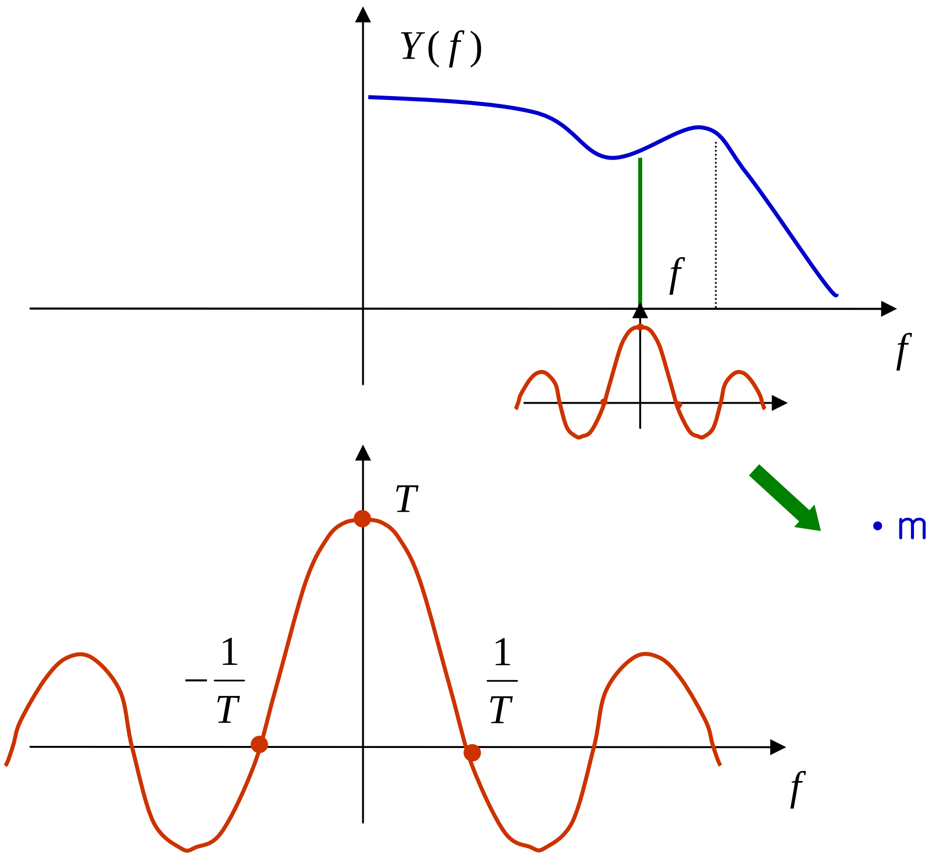
$s = f - \xi$

$\rightarrow \quad ds = df$

$$= \int_{-\infty}^{\infty} y(t)\, U(\xi)e^{j2\pi\xi t}\, d\xi \;= y(t)\, u(t)$$

$$X(f) = U(f) * Y(f) = \int_{-\infty}^{\infty} Y(f - \xi) U(\xi) d\xi \quad ?$$

$Y(f)$



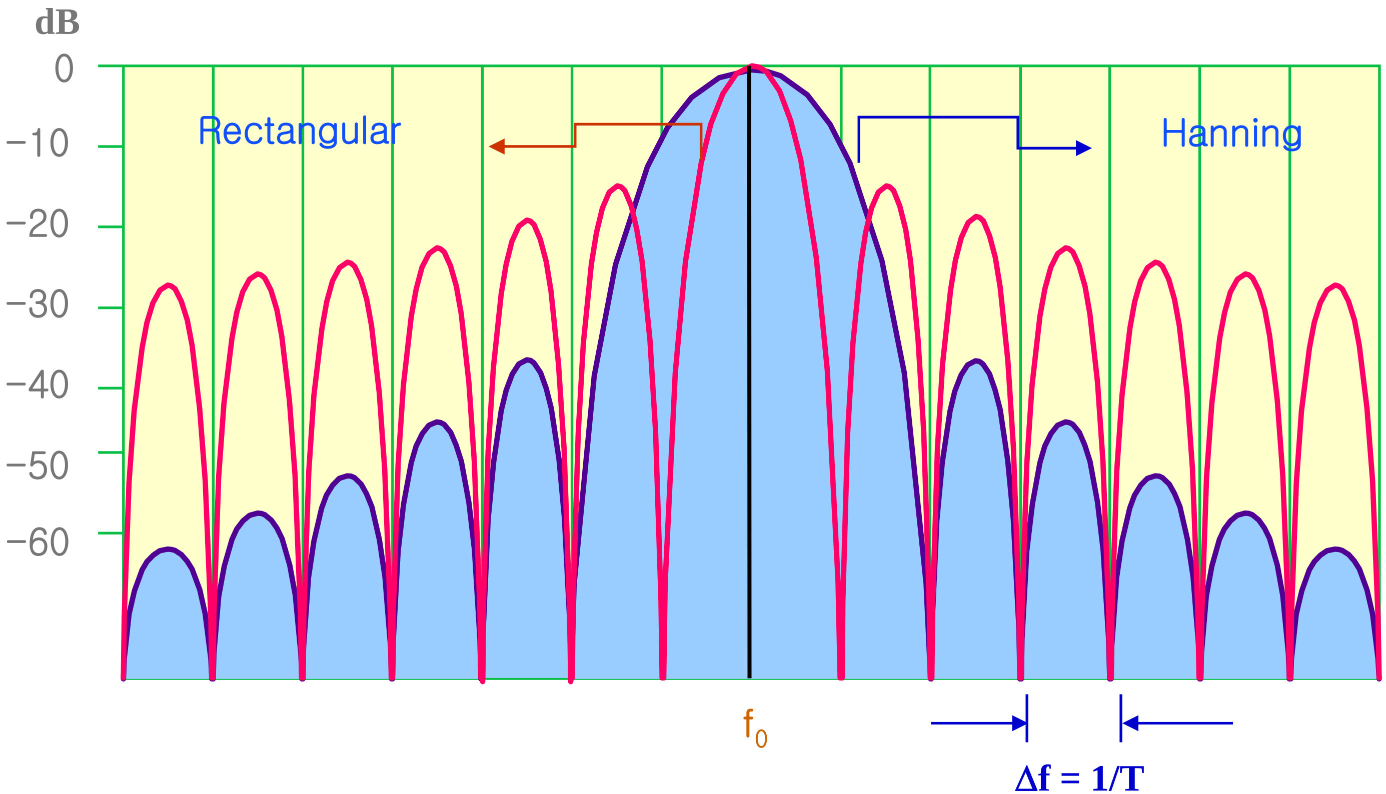
$$U(f) = \frac{T \sin \pi f T}{\pi f T}$$

• multiply and sum

$X(f)$

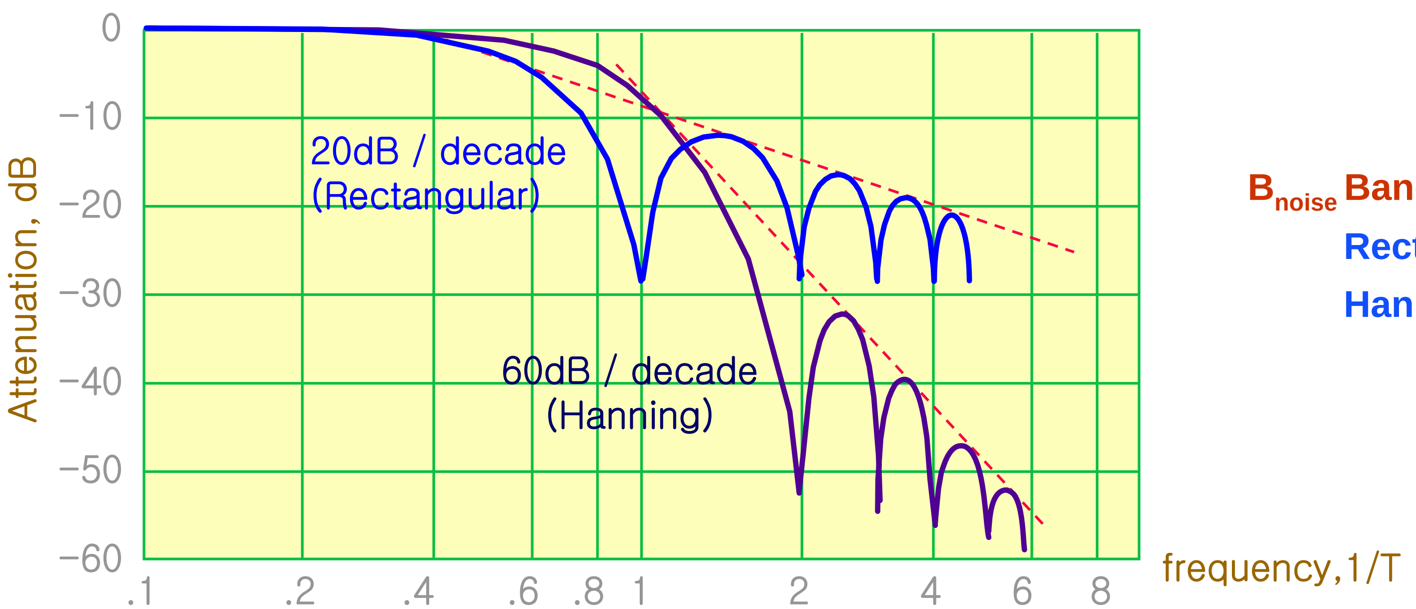
• If u(t) is delta function, X(t) will exactly represent Y(f) .

- Weighting function – Spectra of Rectangular & Hanning Windows



at f_0 Coinciding with one of the FFT Lines

- Weighting function – Filter Characteristics



B_{noise} **Bandwidth**

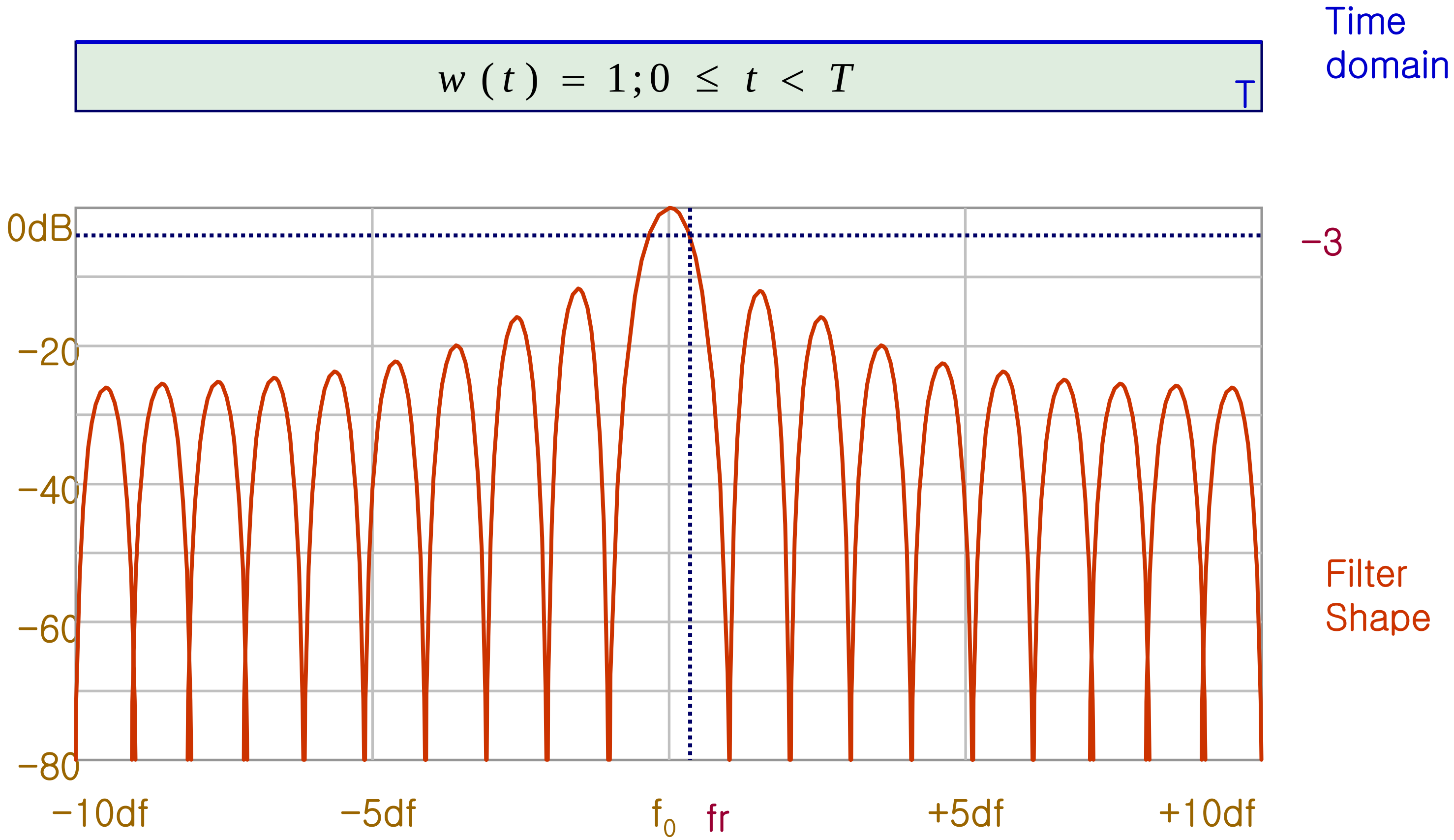
Rectangular : Δ f

Hanning : 1.5 Δ f

At −20 dB

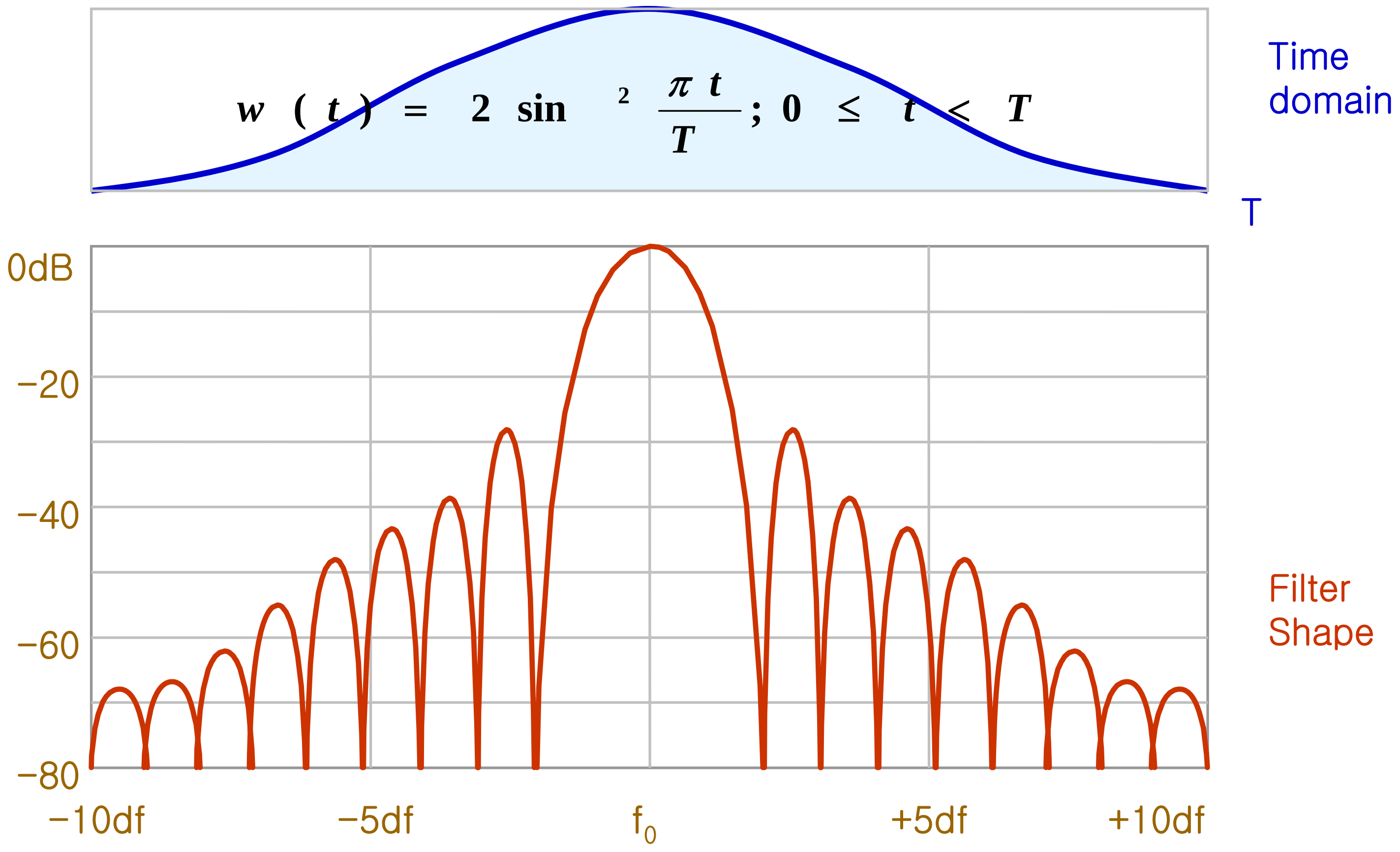
The shape of the Hanning window causes the side lobes to fall off with a rate of −60dB per decade compared to the fall off rate of −20dB for the Rectangular window, making the Hanning window more selective.
However, the more narrow Hanning window has a wide noise bandwidth than has the Rectangular window.
This must be taken into account when calculating the total power of a signal or the power spectral density for random signals.

- Weighting function – Rectangular Weighting



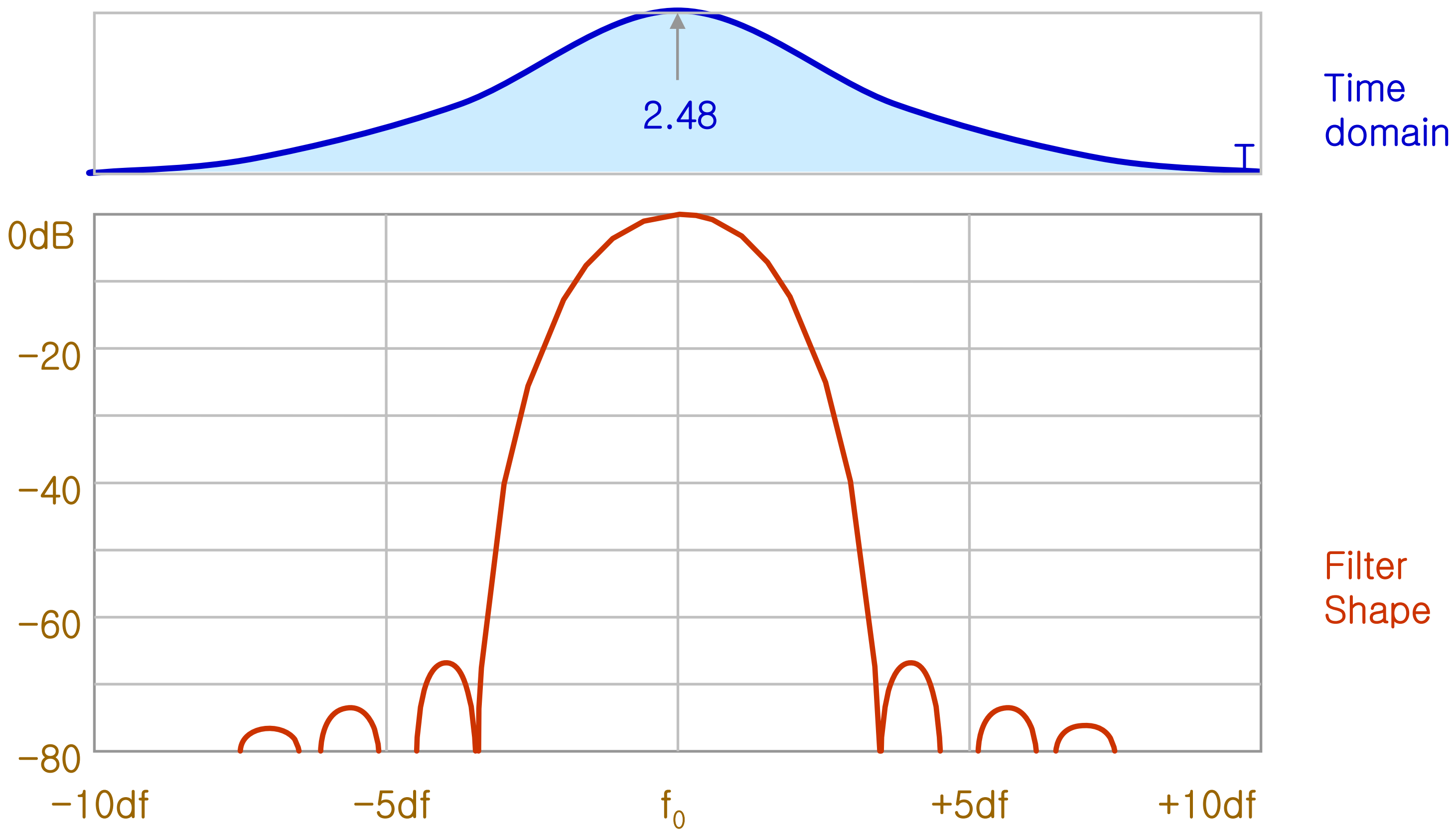
- Weighting function – Rectangular Weighting
  - Uniform window weights all of the time record uniformly
  - Signal which is happened on short time
  - Order tracking
  - Mobility test using pseudo-random excitation signal

- Weighting function – Hanning Weighting

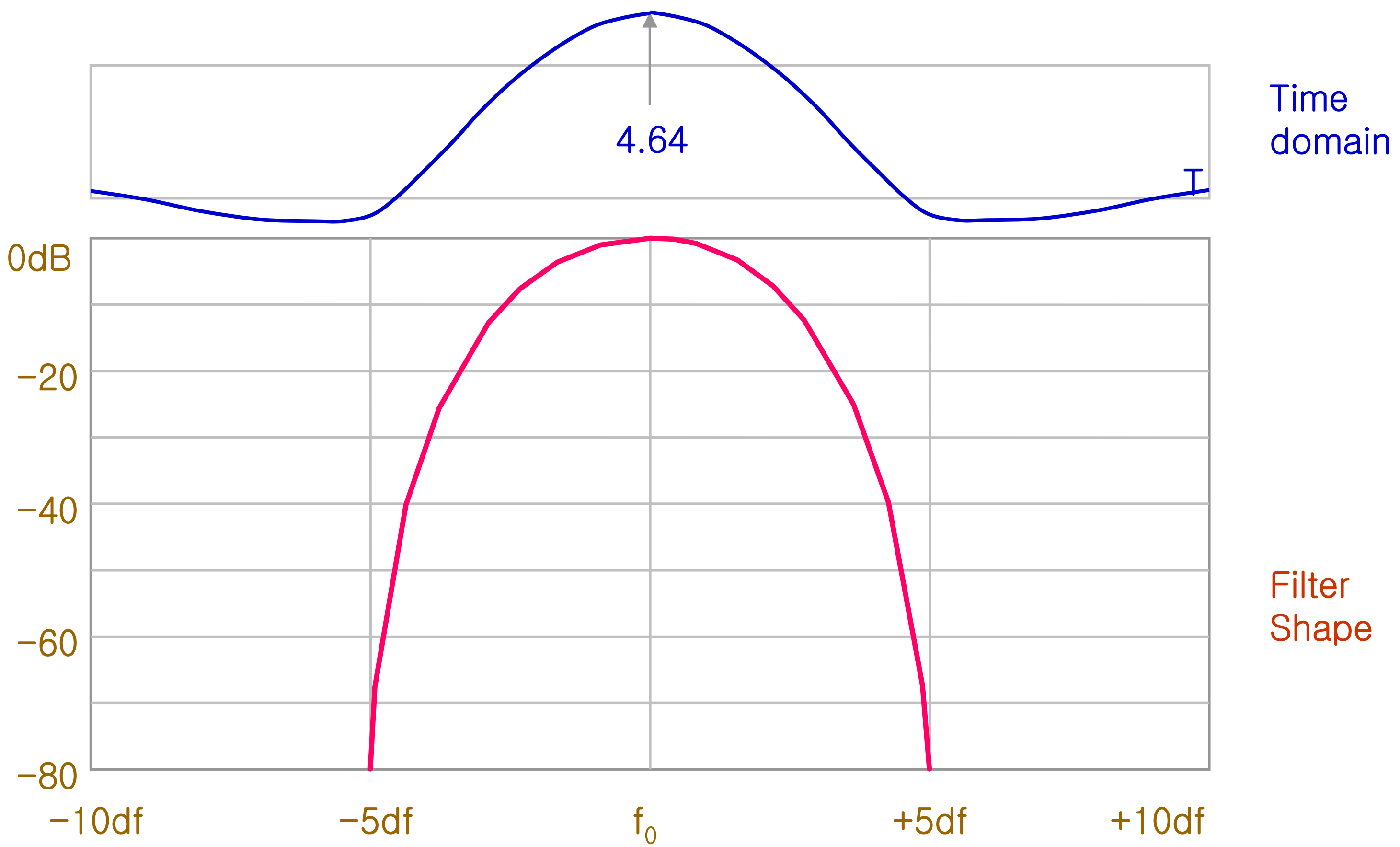


- Weighting function – Hanning Weighting
  - Continuous signal
    - Random noise, periodic signal
  - Modal testing using random excitation signal
  - Using combined with overlap analysis

- Weighting function – Kaiser-Bessel Weighting



- Weighting function – Flat Top Weighting

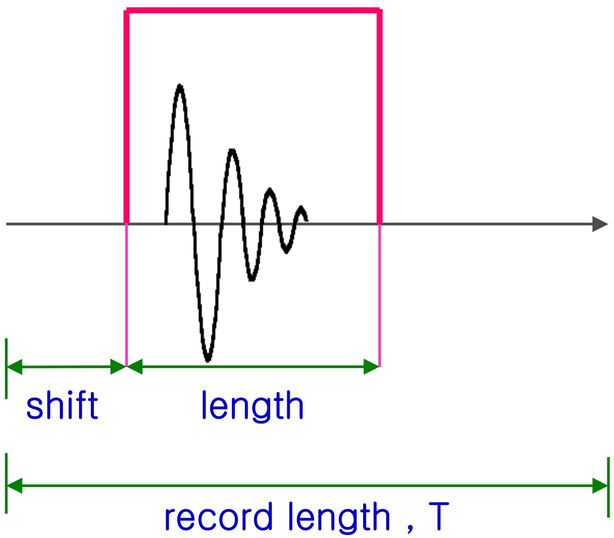


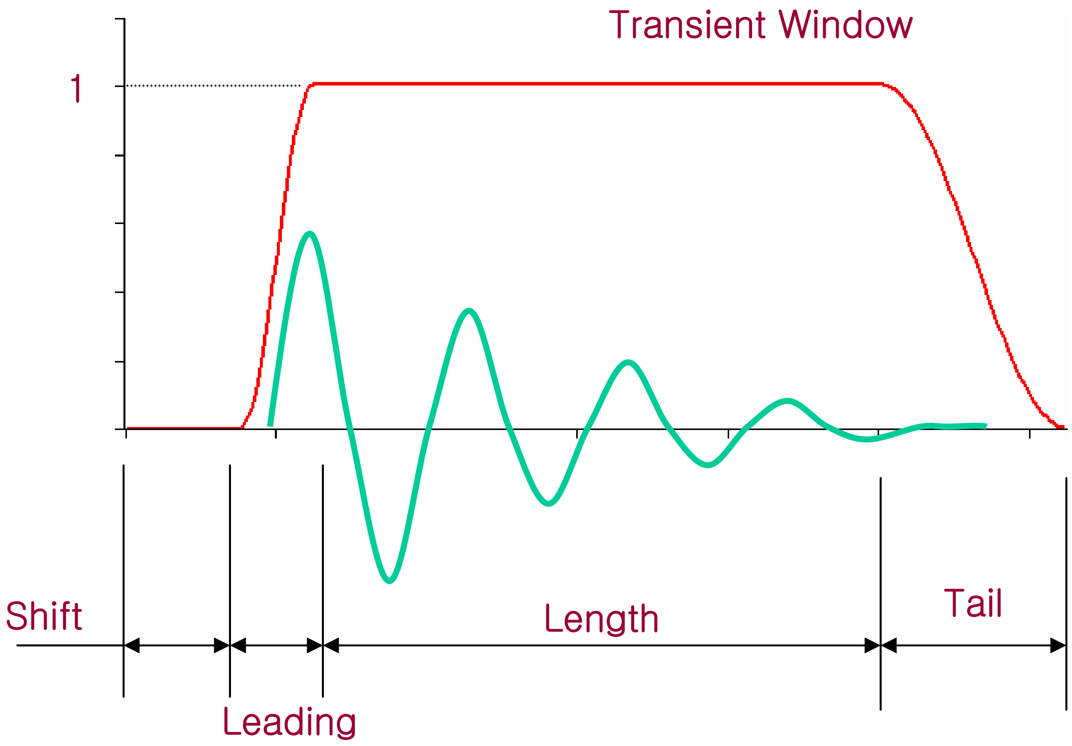
- Weighting function – Flat Top Weighting

$$
\begin{aligned}
w(t) = 1 - 1.93 \cos 2\pi t / T \\
+ 1.29 \cos 4\pi t / T \\
- 0.388 \cos 6\pi t / T \\
+ 0.322 \cos 8\pi t / T
\end{aligned}
$$

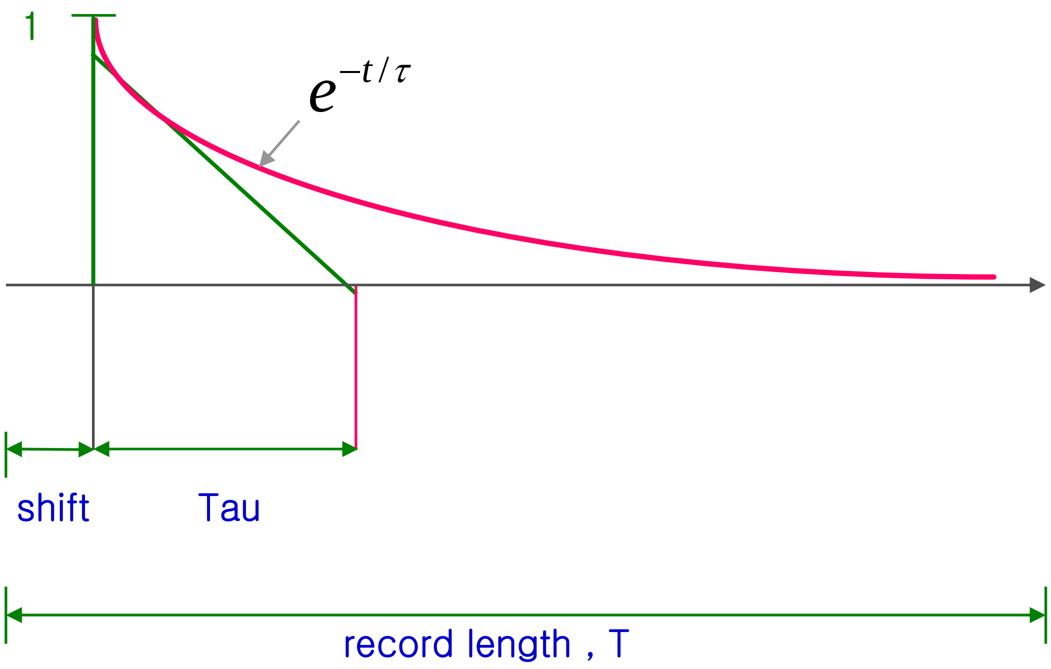
- Small Ripple(> 0.01dB)
- Calibration
- Bad frequency resolution

- Weighting function – Transient Weighting and its example

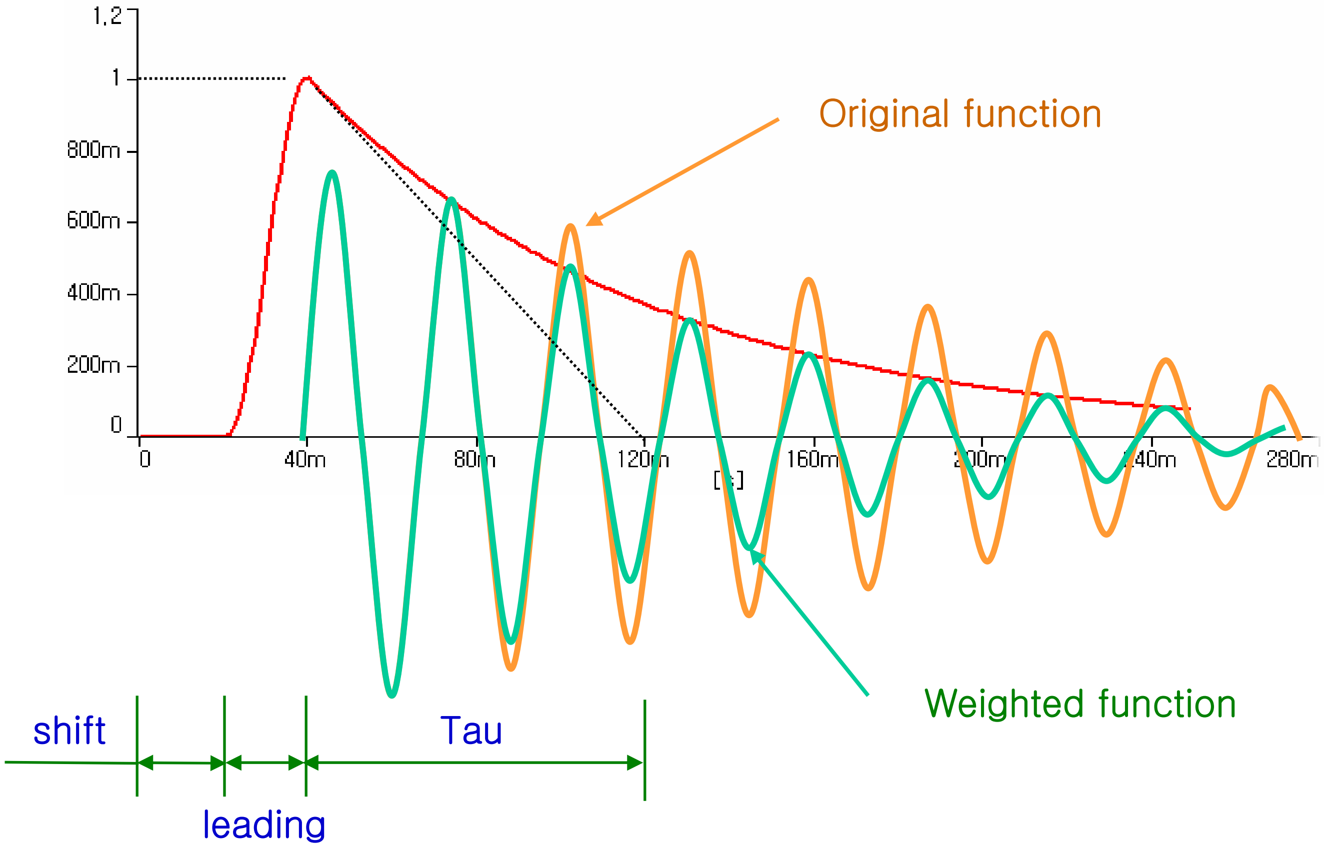




- Weighting function – Exponential Weighting



- Weighting function – Exponential Weighting Example



- Use of Weighting Functions in Signal Analysis

  - Transients:
    - Rectangular : General purpose
    - Transient : Shot transients
    - Exponential : Very long transients
    - Hanning, Overlap : Very long transients

  - Continuous signal:
    - Hanning :

      General purpose.

      Real-Time Analysis
    - Kaiser-Bessel : Two-tone separation
    - Flat Top : Calibration
    - Rectangular : Pseudo random signals

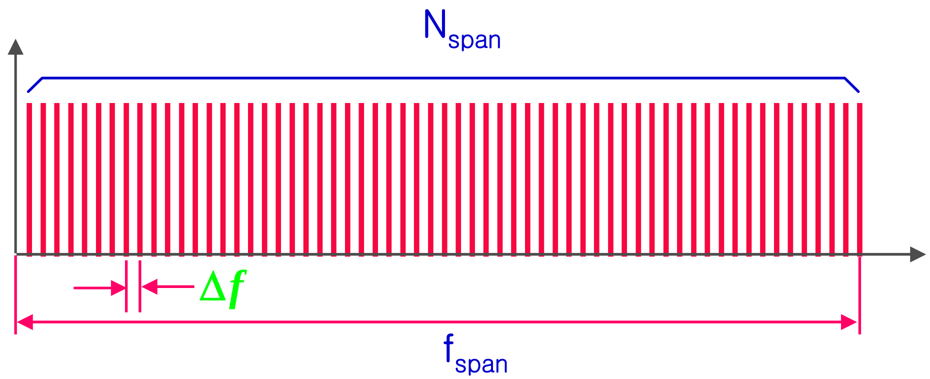
- Use Weighting Functions in System Analysis
  - Impact excitation:
    - Transient & Exponential
  - Random Impact excitation:
    - Hanning
  - Random excitation:
    - Hanning
  - Pseudo random excitation:
    - Rectangular

- How to Avoid the Pitfalls of DFT

Aliasing	cause	Sampling in time
	solve	Anti-aliasing filter ($f_s > 2f_m$)
Leakage	cause	Time limitation
	solve	Increase frequency resolution
Picket fence effect	cause	Sampling in frequency
	solve	Increase frequency resolution

- Weighting Functions Summary
  - Many different windows exist for different purposes.
  - Use of the proper window can reduce leakage and picket fence effect errors.
  - Windows can be regarded as filter.

- Calculating the Real-time Bandwidth



$$f_{span} = N_{span} \times \Delta f = N_{span} \times \frac{1}{T}$$

Real-time requirement : $T \geq T_{AN} \Leftrightarrow f_{span} \leq N_{span} \times \frac{1}{T_{AN}}$

Real-time bandwidth : $f_{real-time} = N_{span} \times \frac{1}{T_{AN}}$

- Use of Real-time Analysis
  - Real-time Analysis is not necessary for :
    - Stationary signals
    - Transients (if recording is real time)
  - Real-time Analysis is necessary for :
    - Non-stationary signals
      - Examples:

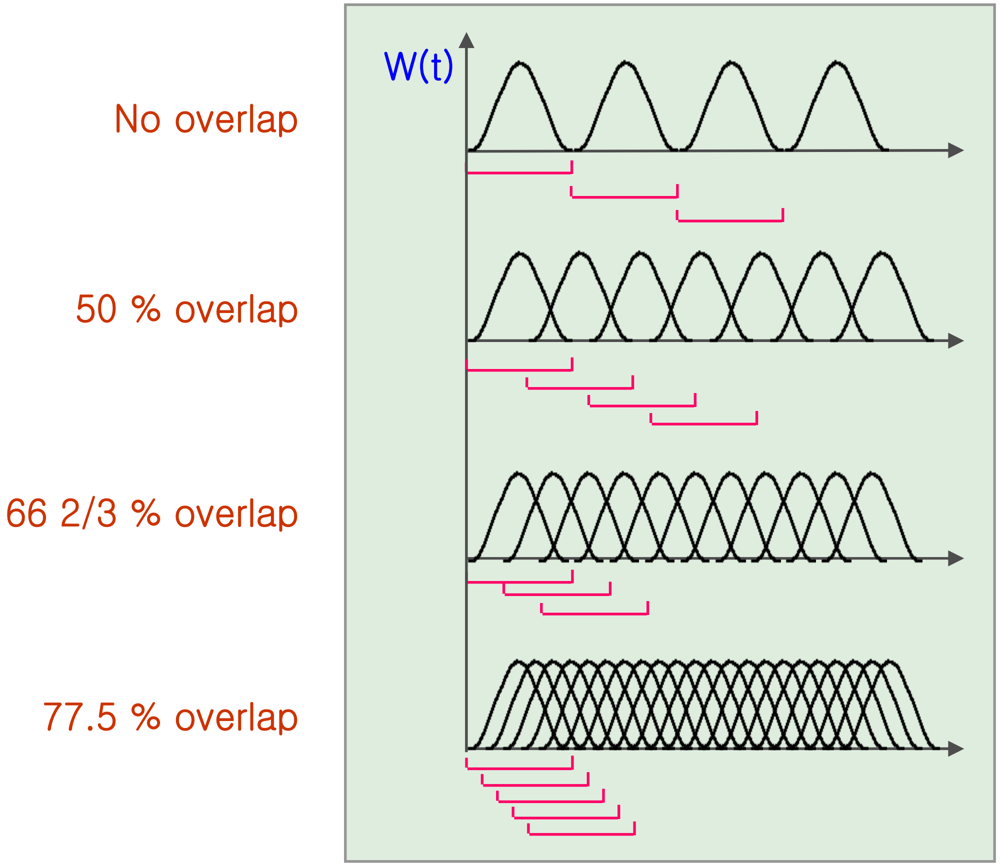
        Fast run-up/down tests

        Reverberation time measurements

        Vehicle by-pass noise

        Fly-over noise

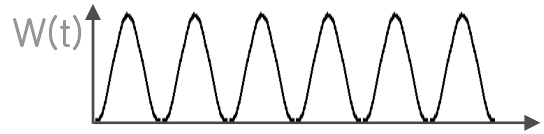
- Overlap Analysis with Hanning Weighting



When using other weighting functions than the Rectangular weighting, the condition $T \geq T_{an}$ is not sufficient to avoid loss of data and thereby possible loss of valuable information.
To illustrate this, Hanning weighting is used here as an example. If the time records do not overlap, parts of the signal will not be included in the average. To avoid this, overlap processing can be used.
For Hanning weighting 50% overlap, 66 2/3 % overlap or 75% overlap are often used. But this requires that the conditions $T \geq 2\ T_{an}$, $T \geq 3\ T_{an}$, $T \geq 4\ T_{an}$ respectively are fulfilled.

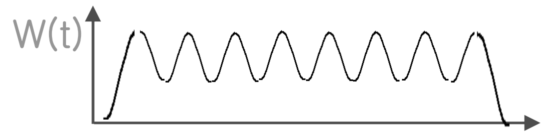
- Overlap (Hanning Weighting)

No overlap



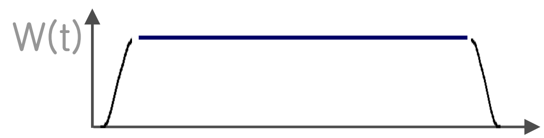
$$(1-\cos x)^2 = 1 - 2\cos x + \cos^2 x$$

50 % overlap



$$\frac{1}{2}\left[(1-\cos x)^2 + (1+\cos x)^2\right] = 1 + \cos^2 x$$

66.7 % Overlap



$$\frac{1}{3}\left[(1-\cos x)^2 + (1-\cos x(x-\frac{2\pi}{3}))^2 + (1-\cos(x-\frac{4\pi}{3}))^2\right] = 1.5$$

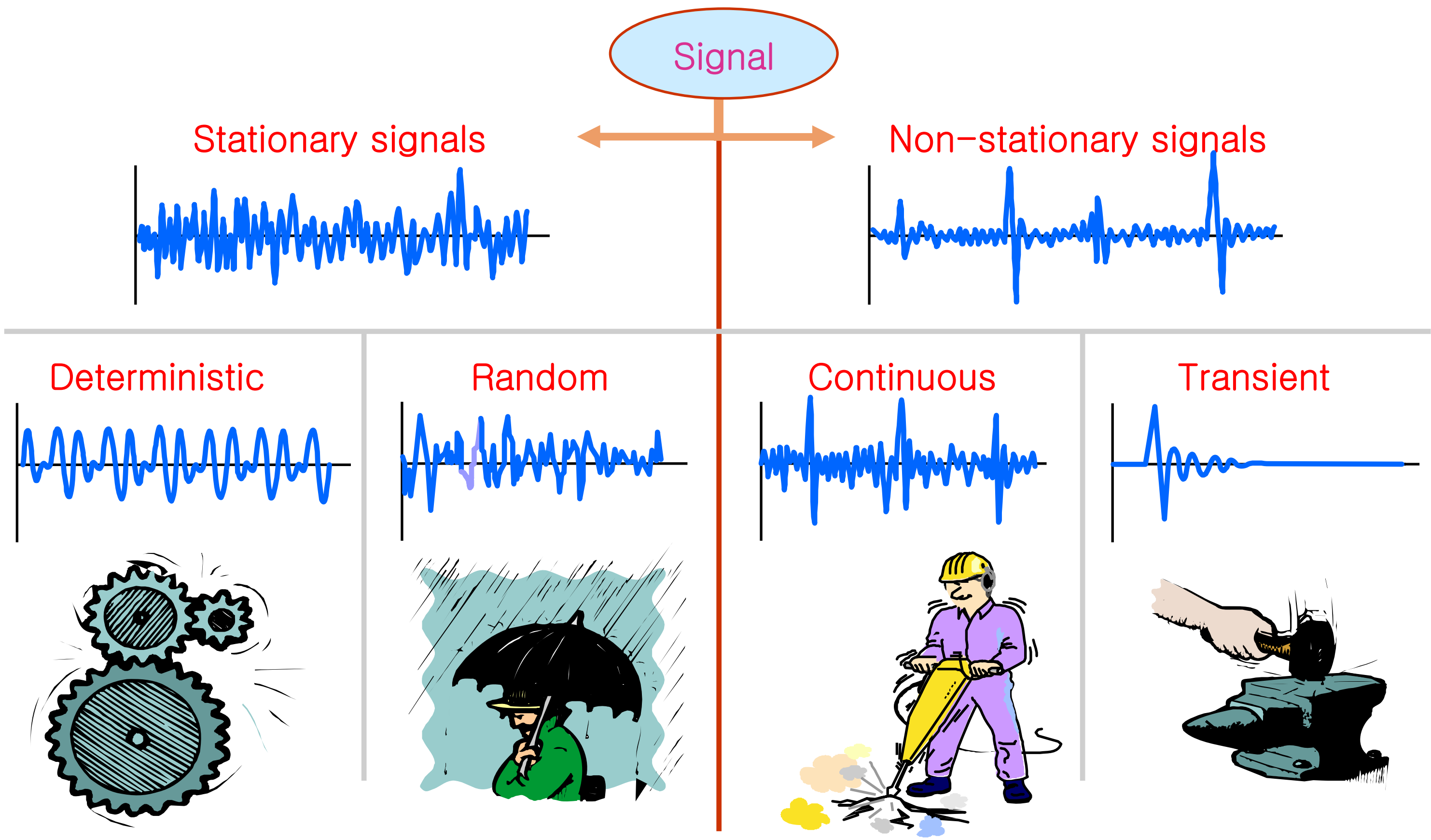
- Application of Overlap Analysis

  - Overlap analysis is necessary in order to avoid loss of data when using other windows than Rectangular
  - Analysis with a certain accuracy of a random signal is faster with Hanning weighting and 50% overlap than with Rectangular weighting
  - Equal weighting of all time data is obtained with Hanning weighting and overlap of 2/3(66.6%), 3/4(75%), 4/5(80%)....

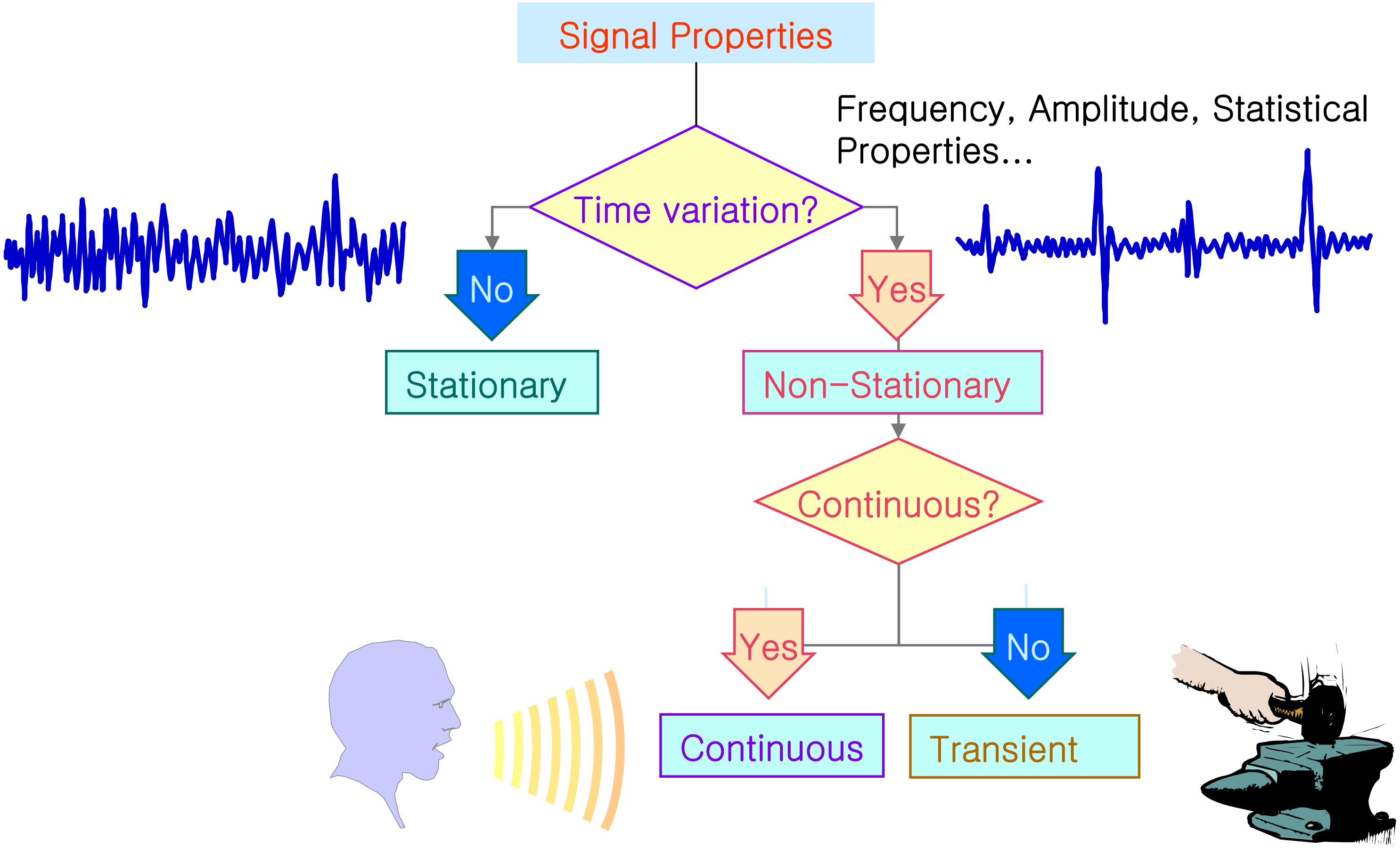
Stationary
- Amplitude correction

Transient
- Continuity of time data
- Reduce measurement time

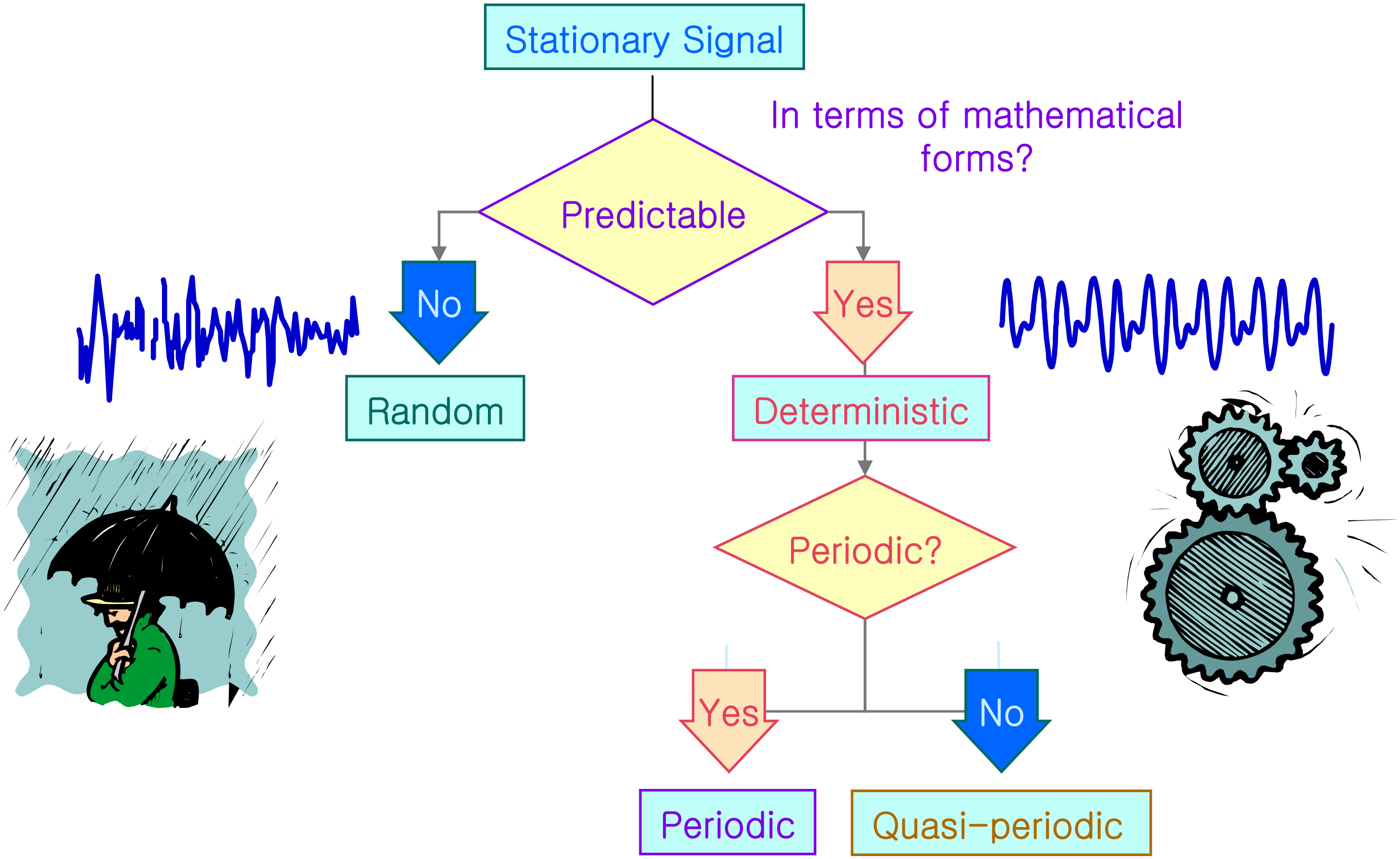
- Signal Types



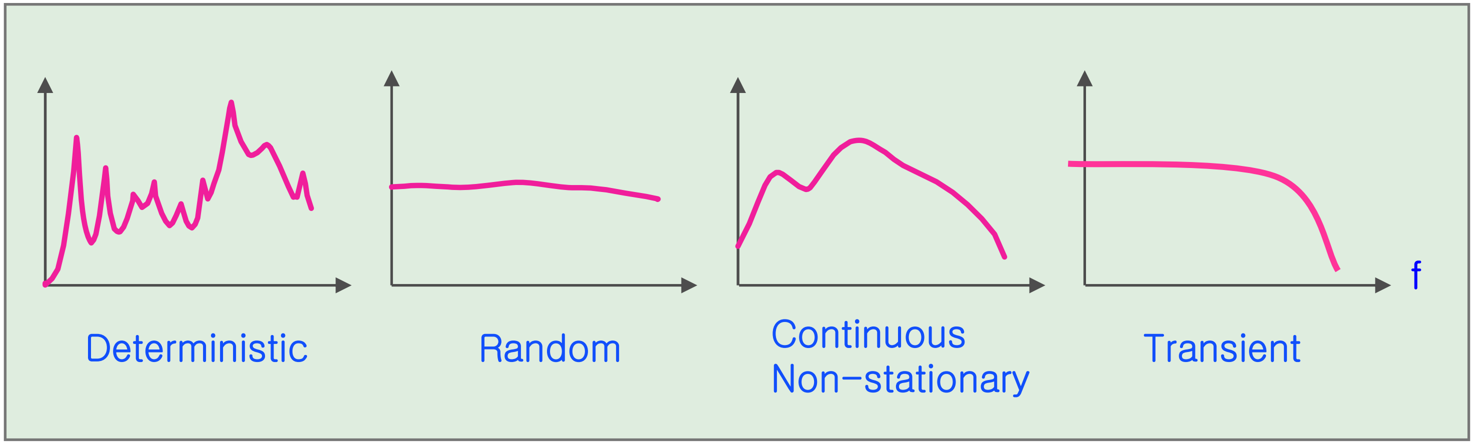
- Stationary & Non-stationary

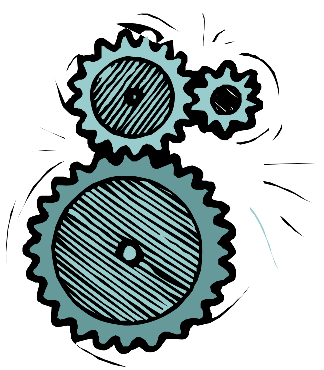


- Stationary Signal

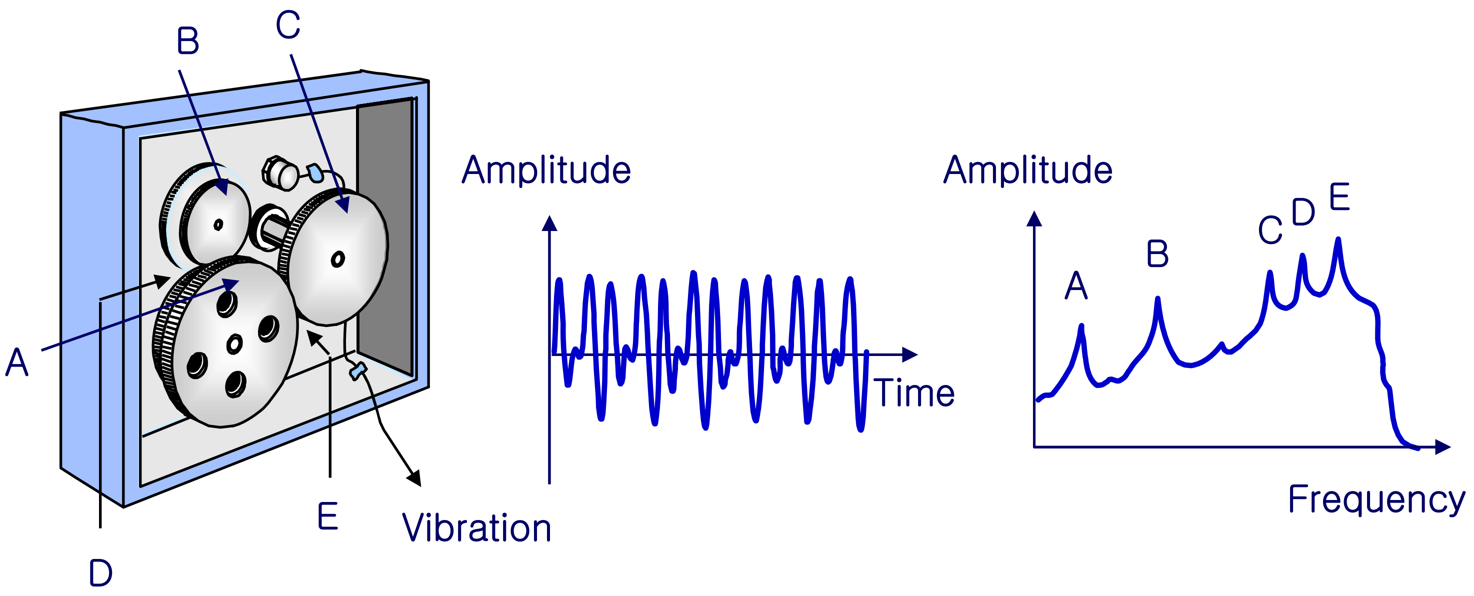


- Signal Shape

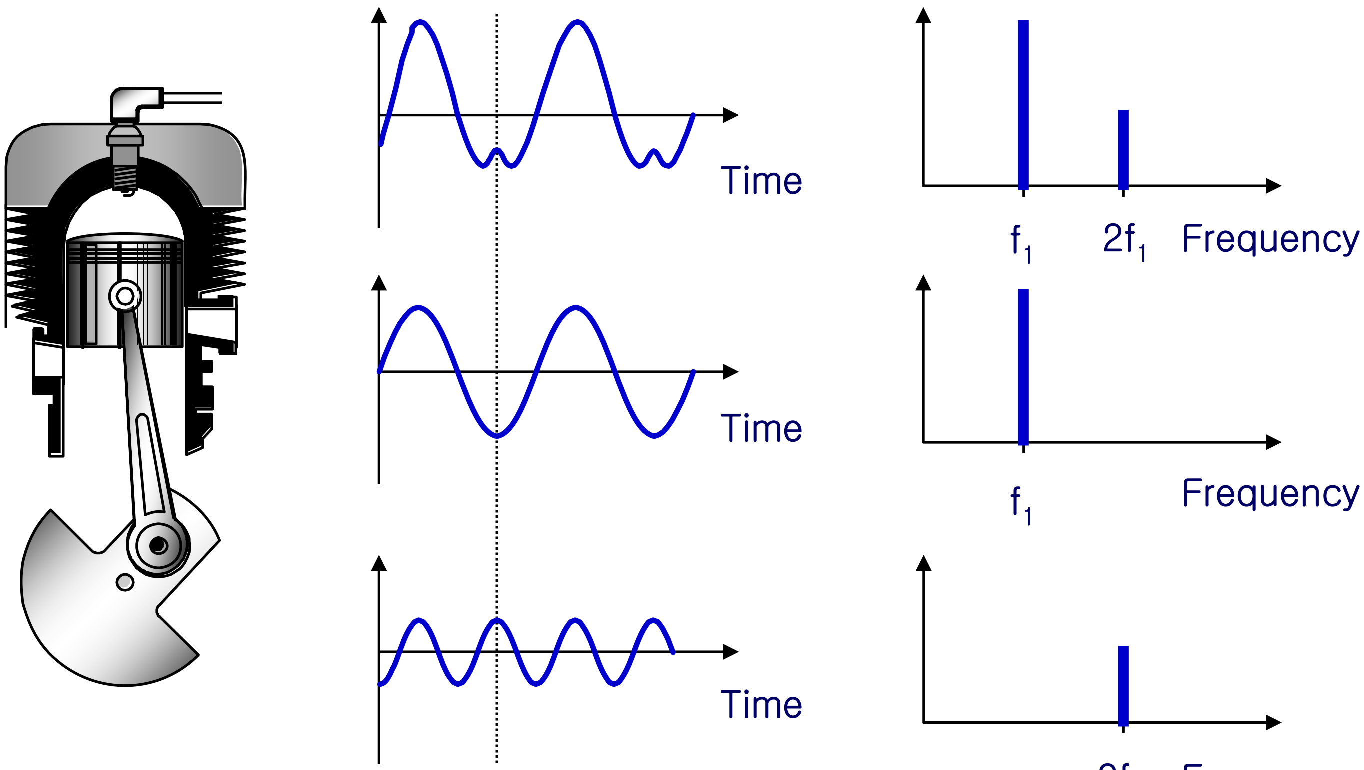




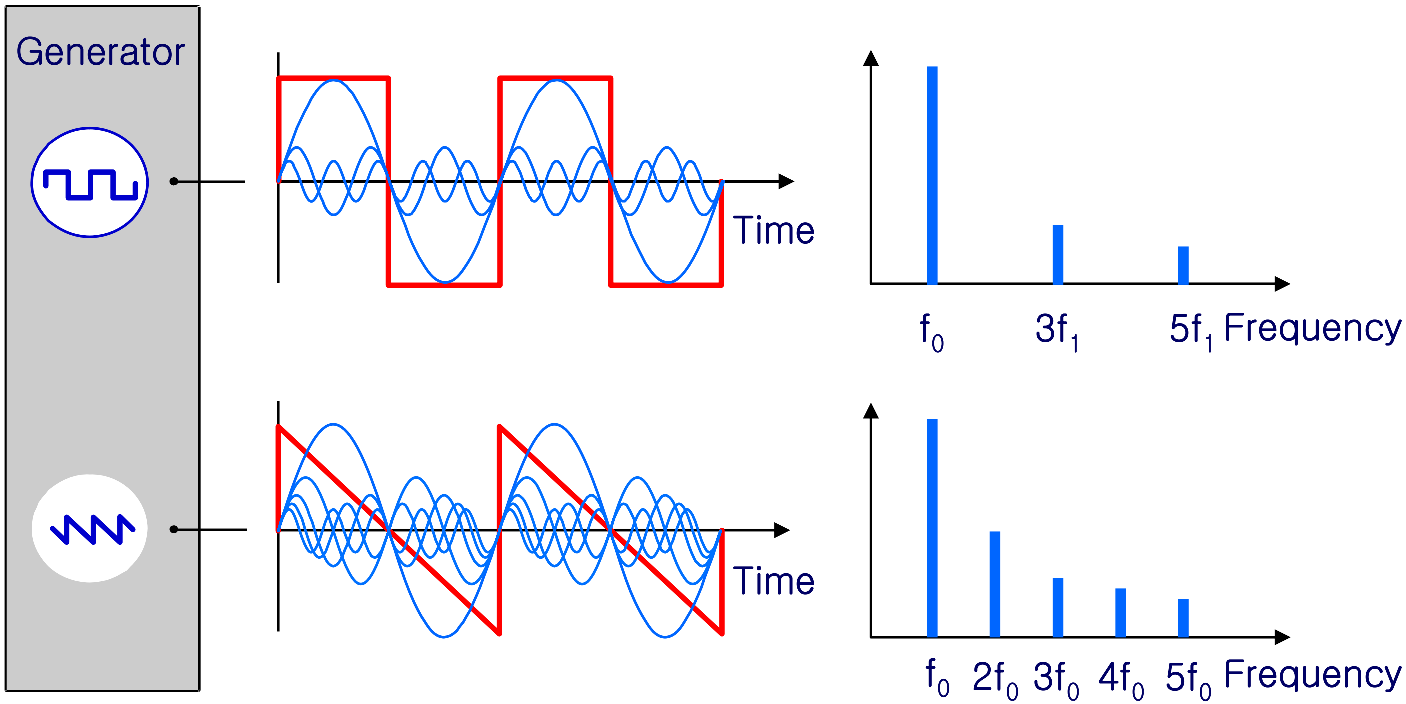
- Deterministic Signals



- Deterministic Signals and Harmonics

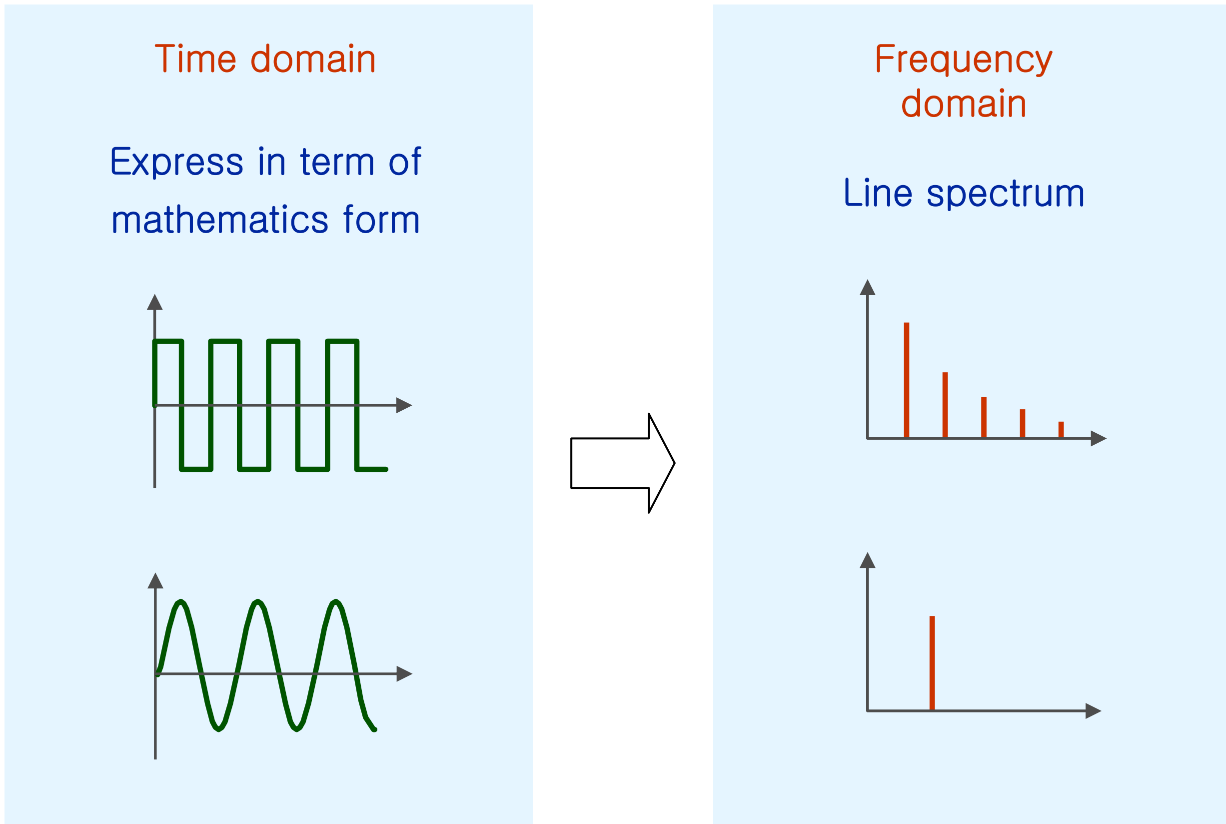


- Harmonics

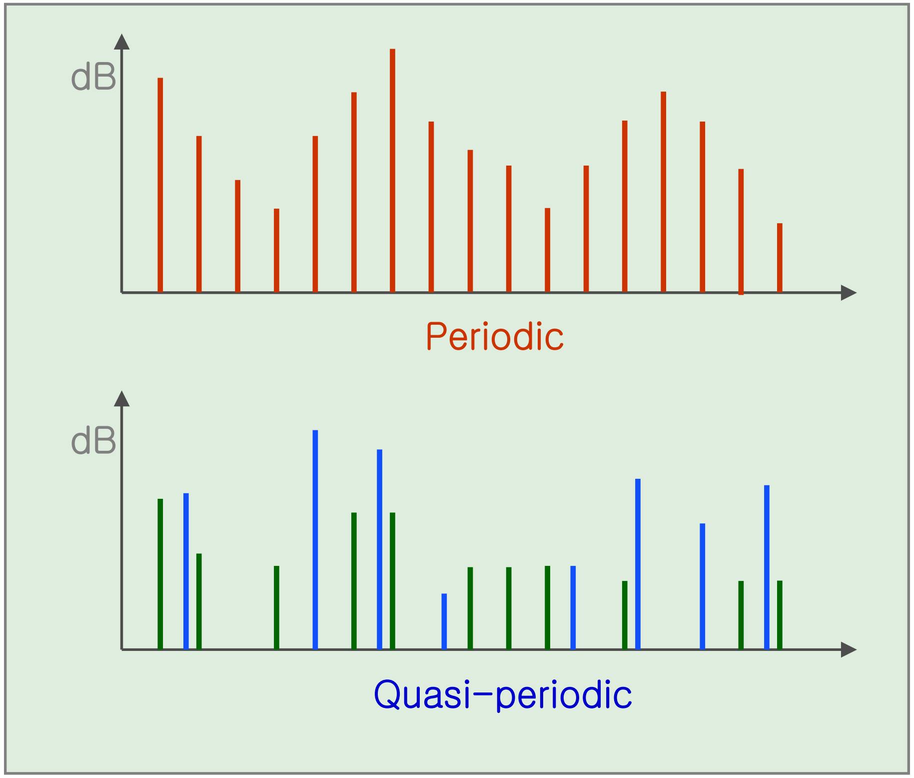


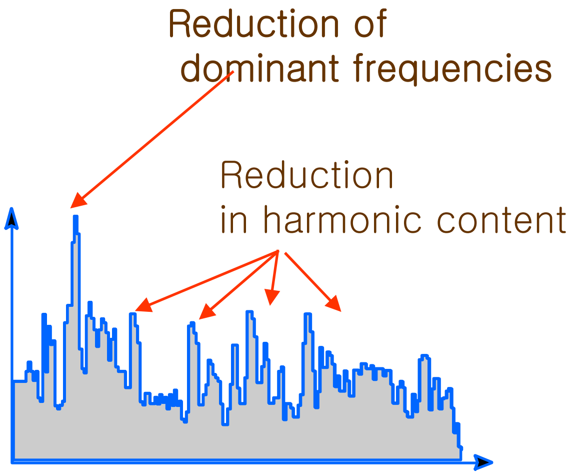
Harmonic component are related fundamental frequency.

- Deterministic Signal



- Spectra of Deterministic Signals





- Deterministic Signal Analysis

  - Fourier series expression ⇒ Line spectrum

$$x(t) = \sum_{n=0}^{\infty} X_n \cos(2\pi f t - \theta_n)$$

  - If filter bandwidth is smaller than line spacing, measurement value is independent of filter bandwidth.
  - Measuring unit : Power, RMS

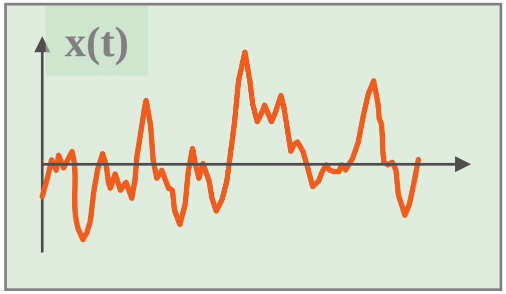
- Random Signal
  - It can be expressed in term of probability value not math form
  - Data set which is collected has unique value
  - Produce continuous spectra
  - Analysis method

    Frequency domain

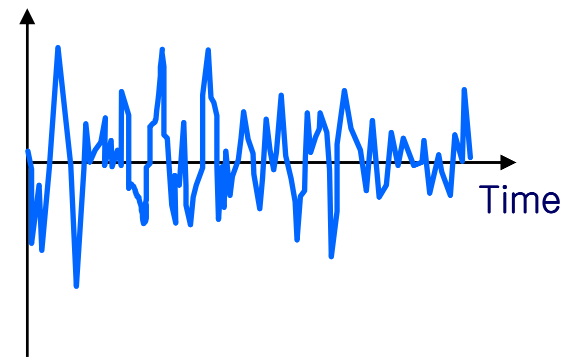
    - Power Spectrum, Cross Spectrum

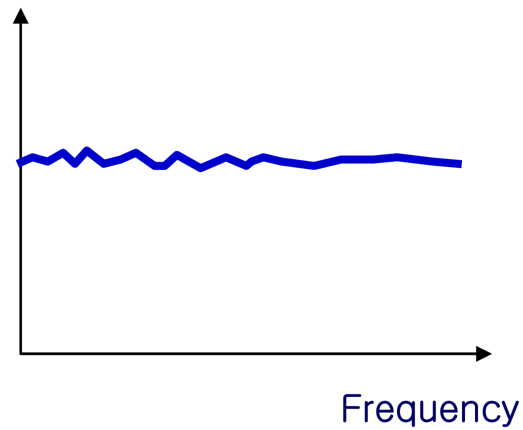
    Time domain

    - Mean Value, RMS, Probability Density Function, Auto-correlation, Cross-correlation

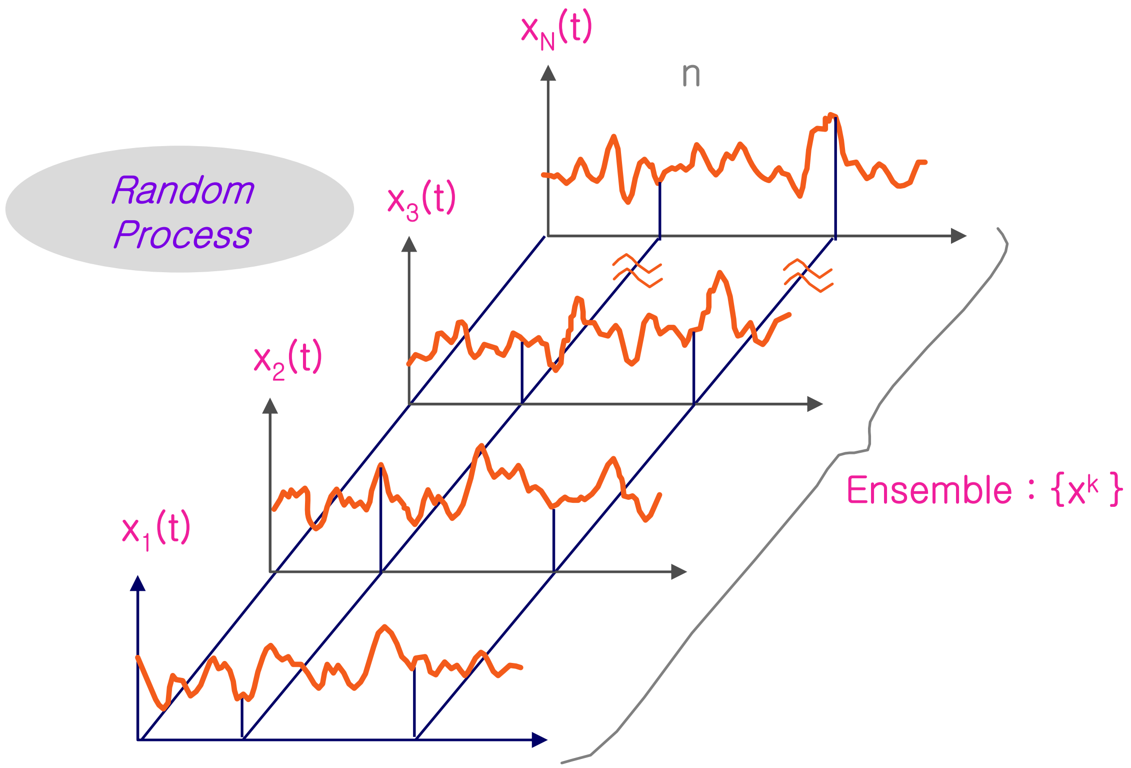


- Random Signals





- Stationary Random Signal



- Stationary

  - Ensemble average

$$\mu_x(t_1) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N} x_k(t_1) = \mu_x(t)$$

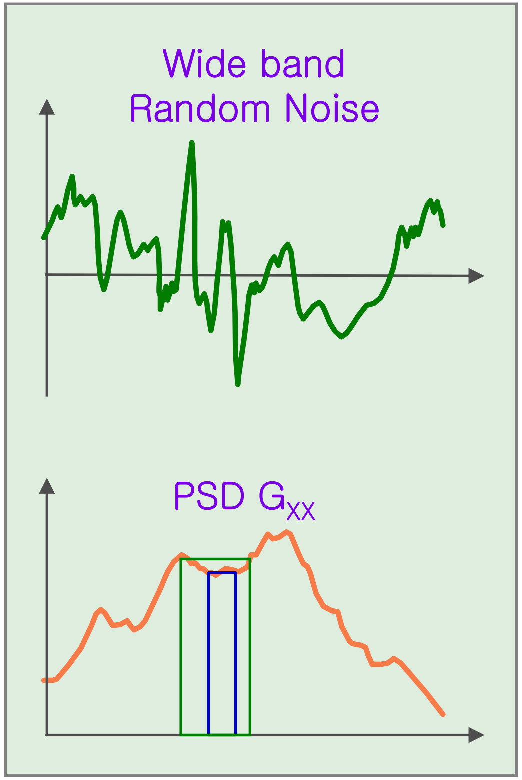
$$R_{xx}(t_1, t_1+\tau) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N} x(t_1)x_k(t_1+\tau) = R_{xx}(t,\tau)$$

stationary

3.5 Measurement for Signal Types

- Random Signal Analysis

Since Random signals have continuous spectra, the amount of power transmitted by the analyzing filter will depend on the filter bandwidth.



Power is increased according to filter bandwidth

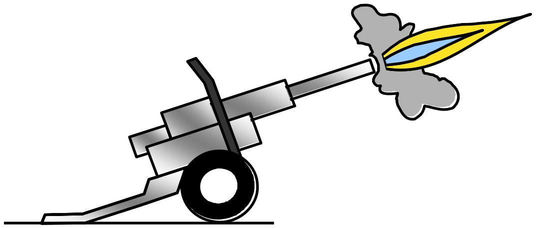
Normalize filter bandwidth

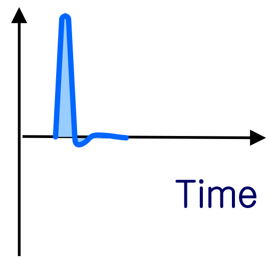
$$PSD = \frac{Power}{Bandwidth}$$

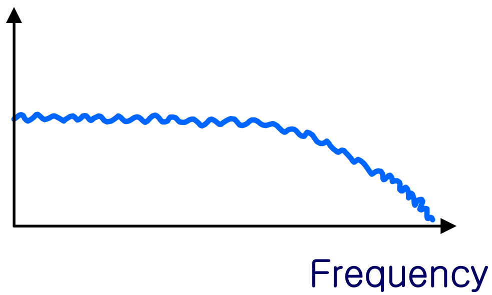
Power Spectrum Density : $Unit^2$ /Hz

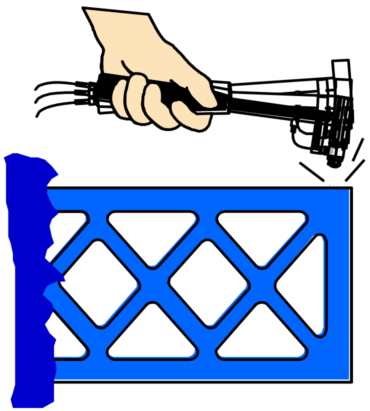
3.5 Measurement for Signal Types

- Impact-Impulse-Shock Signals



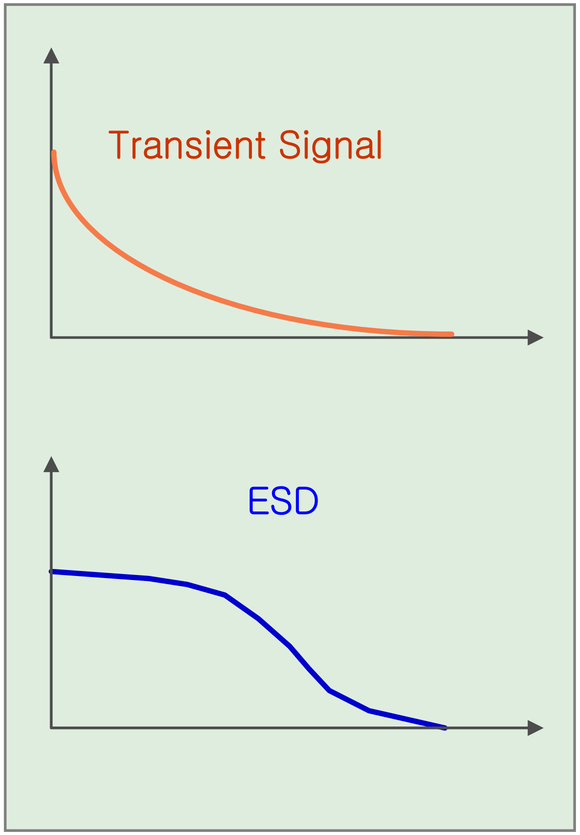


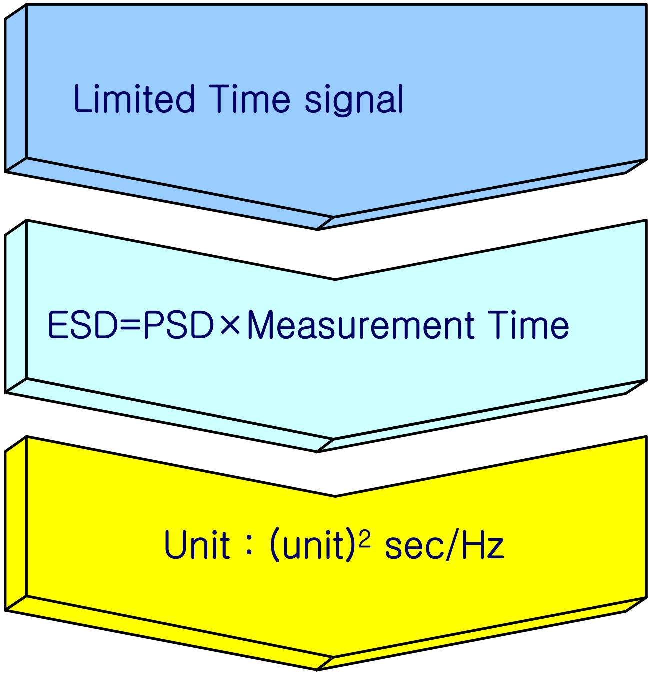




Mechanical shock is a short burst of vibratory energy.
If the shock is infinitely short it will also have a frequency spectrum which is distributed continuously with frequency. Since a shock will always have a finite length its frequency spectrum will be limited to a band of frequencies.

- Transient Signal Analysis

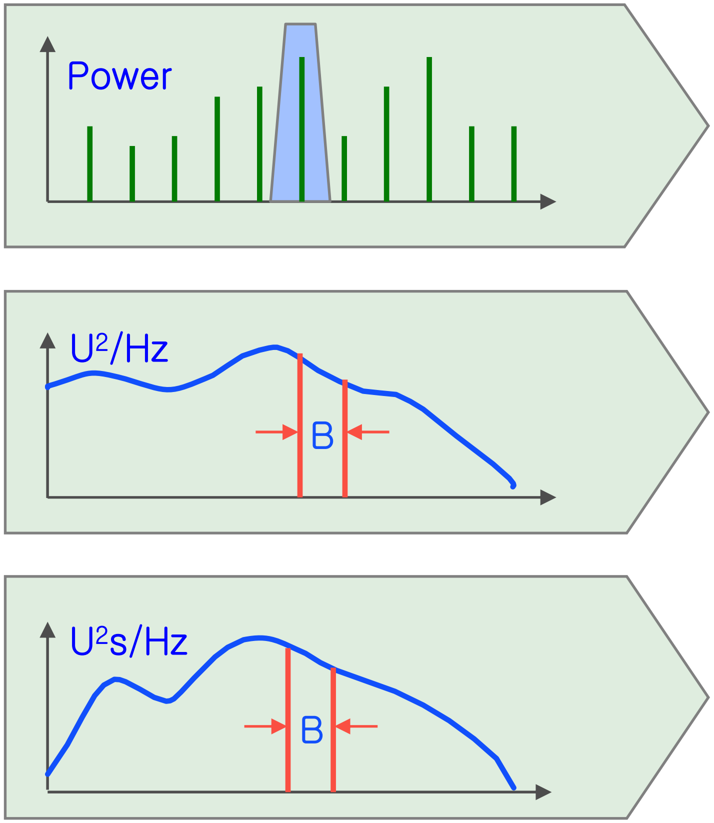




Energy Spectrum Density

3.5 Measurement for Signal Types

- Transient Signal Analysis



Deterministic
RMS, Power

Random
PSD = U^2 /Hz

Transient
ESD = $U^2 s$ / Hz

5. Measurement for Signal Types

- Stationary & Non-Stationary Signals
  - Stationary

    Vibration from rotating machines(steady state)
  - Non-Stationary

    Characteristics changing
    - Frequency / Amplitude / Statistical properties

    Examples :
    - Run-up / cost-down tests of machinery(long term)
    - Analysis pass-by / fly-over noise
    - Speech analysis
    - Reverberation time measurements
    - Reciprocating machines
      - Short term : non-stationary signal
      - Long term : stationary signal